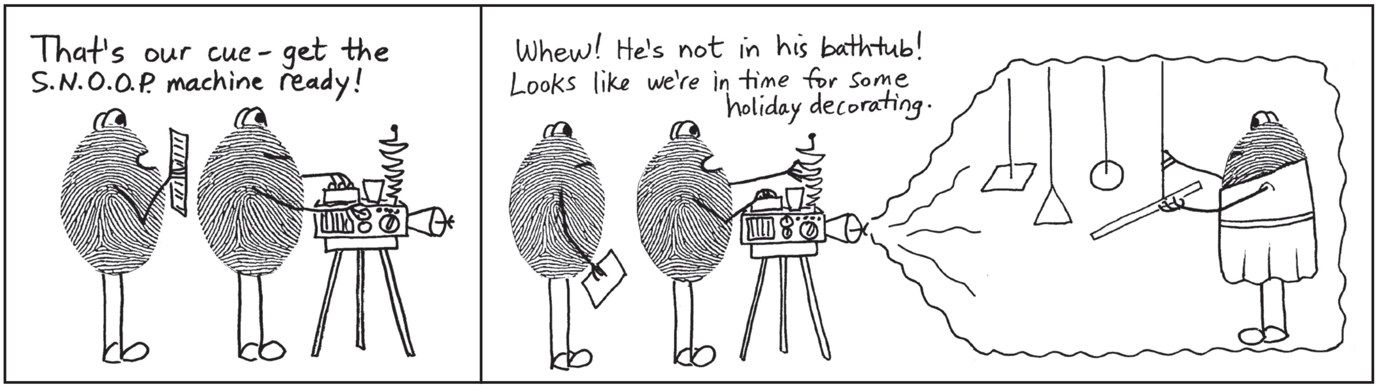
Adventure 1: Balancing with Archimedes

Our adventure begins about 2,200 years ago in the ancient Greek city of Syracuse, on the island of Sicily. (On a map, Sicily looks like the "ball" being kicked by the "boot" of Italy.) A brilliant mathematician named Archimedes *(ar-kih-ME-deez)* is experimenting with the way things balance. Archimedes wasn't the first person to think about how things balance, but he was the first to propose the idea that we now call "center of gravity." In Archimedes' day the word "gravity" had not yet been invented, so he didn't use that word.

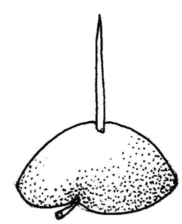


Archimedes was interested in finding a mathematical way of describing how and why things balanced. It was easy to find the balance point for a wooden rod, but what about a flat shape such as a triangle or a trapezoid or a parallelogram? His final conclusions were written in a book called *On The Equilibrium of Planes*. ("Equilibrium" means "balancing," and in geometry, a plane is a flat surface.)

Finding the balance point ("center of gravity") of something can be quite important in our everyday lives. When you hang a picture on the wall, if you get the hanger off-center the picture will look crooked. If you are making a spinning top and don't get the handle perfectly centered, the top won't spin correctly. You can probably think of more examples just by looking at things around you right now.

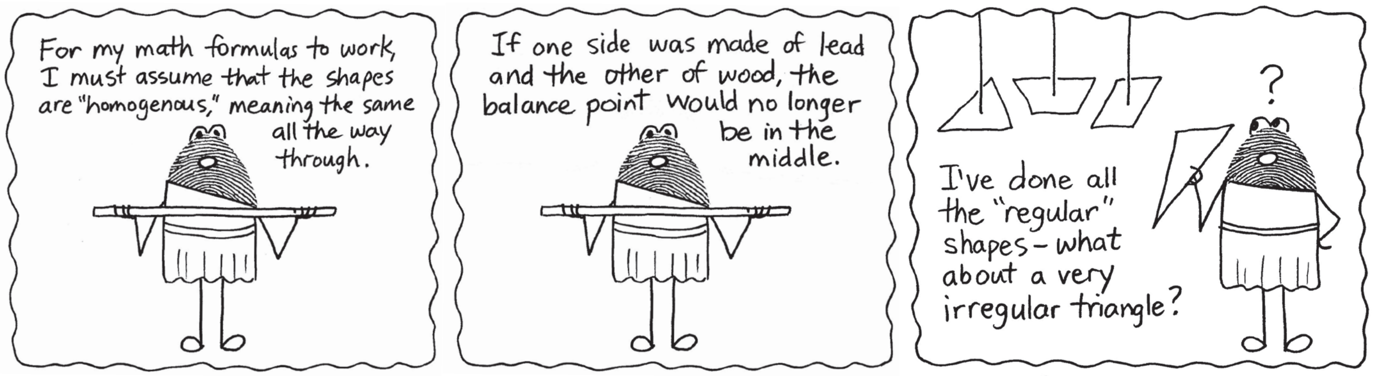
You've probably used a ruler to find the center of a line or a square for a math assignment, or perhaps the center of a board for an art or craft project. Rulers are the most common tool for finding centers. But how accurate are your measurements? In this first activity, you will test your acumen (skill) with a ruler. Can you measure accurately enough to find the balance point for a rectangle? Sounds easy, but is it? Take a few minutes to do the following activity. (Archimedes must have done many experiments like this.)

ACTIVITY 1.1: Use a ruler to find the center of a rectangle, then test your results



You will need: *a ruler, a pencil, scissors, a piece of heavy card stock paper (or thin cardboard such as the side of a cereal box), a toothpick, half an apple (or potato), and a paper towel or paper napkin.*
Stick the toothpick into the apple (or potato or other firm object) to make a "balance stand."

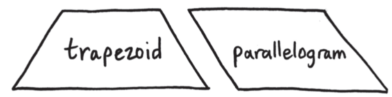
1) *Use the ruler to draw a square on the piece of card stock or cardboard. The square can be about the size of your palm, or a little larger if you wish. (You can get help making a perfectly square corner by tracing the corner of a piece of paper.) Cut out the rectangle.*
2) *Use a ruler to find the center of the rectangle and mark it with a dot. The easiest way to do this is simply to align the ruler so that it connects two opposite corners, then draw a line. Do this to the other pair of opposite corners. The point where these lines cross is the center.*
3) *Place your balance stand on the corner of a table. This will allow you to look at the underside of the rectangle. Or, place the balance stand on top of something tall (but stable) that is sitting on the table.*
4) *Place the center dot of your rectangle right on the point of the toothpick, then let go. Does it balance? If not, move it around until you get it to balance. How far off were you?* (NOTE: **DON'T** prick a hole in your shape using the sharp point of the toothpick! In fact, you can use scissors to trim off the tip of the toothpick so it is not quite so sharp.)



Here is another activity where you can do what Archimedes did. (We are skipping circles because it is pretty obvious that the balance point of a circle is at its center. If you draw a circle using a compass, you automatically get the balance point—the place where you stuck in the pointy part of the compass.)

ACTIVITY 1.2: Find the balance point of trapezoids and triangles

(A trapezoid is a 4-sided figure with at least two parallel sides. A parallelogram is a type of trapezoid.)



You will need: *a ruler, a pencil, scissors, your balance stand, and another piece of card stock or thin cardboard*

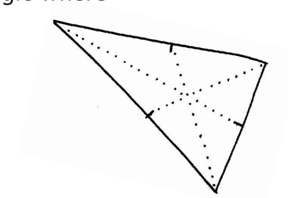
NOTE: If you are working in a group and are short on time, you can have each person in the group make one type of shape, then share them around for balancing.

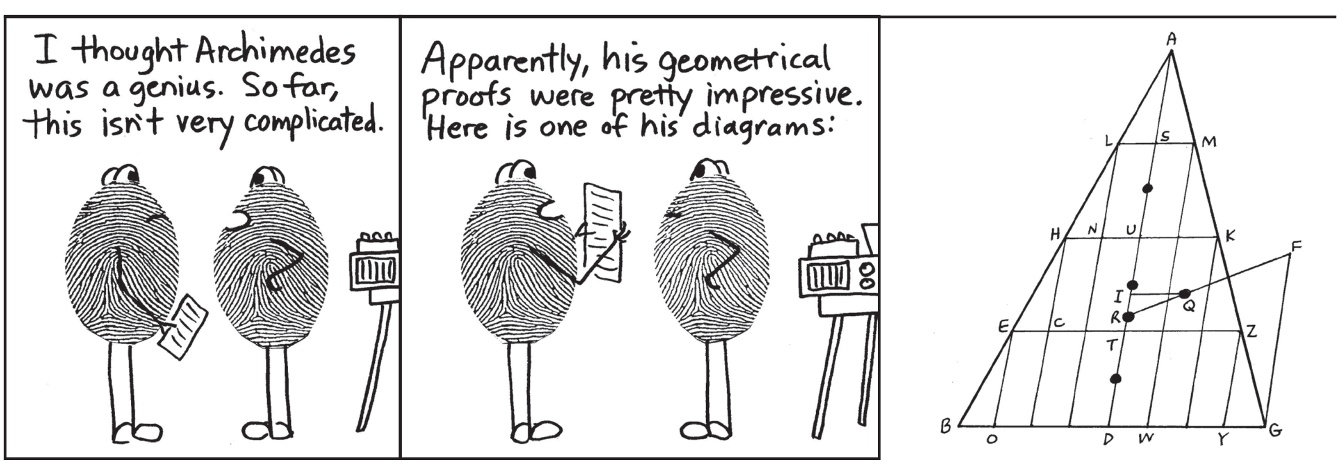
TRAPEZOIDS:

1) *Draw a rectangle, then turn it into parallelogram (which is a special type of trapezoid) by slanting two ends. Make sure the slanted ends are exactly parallel. Then draw another rectangle and turn it into a trapezoid by making those slanted ends into lines that are NOT parallel.*
2) *Cut out your shapes.*
3) *Try the technique you used in Activity #1, where you made lines from the corners that crossed in the middle. Is this the balance point for a parallelogram? For the trapezoid?*
4) *Now try using the ruler to determine the middle of each side. Connect the mid-points of opposite sides. Put a dot at the point where these two lines cross. Is this a better balance point?*

TRIANGLES:

1) *Draw some triangles on your card stock or cardboard. Make one "right" triangle (that has a 90-degree "square" corner), an isoceles triangle (two sides the same length, so it is symmetric), and one "scalene" triangle where each side is a different length. (Archimedes is holding a scalene triangle.)*
2) *Cut out your triangles.*
3) *Use the ruler to find the center of each edge. Mark it with a small dot.*
4) *Now use the ruler as a straight edge to make a connecting line between each center dot and the angle that is directly opposite to it. Make a dot at the place where all three lines cross.*
5) *Place each triangle on the balance stand and see if it will balance on that center dot.*





The words that go with that diagram are even more impressive. Here is a sample:

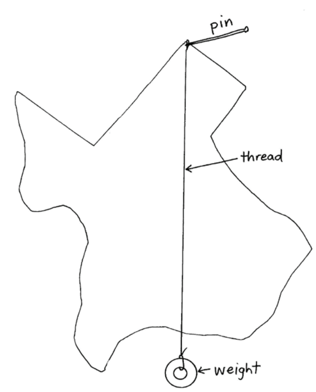
Let there be a triangle, ABG, and let AD in it be against the middle of base BG. One must prove that the "center of weight" of ABG is on AD. For otherwise, still, if it is possible, let it be Q, and let QI be drawn through it parallel to BG. And, in fact, what remains of the repeatedly bisected DG will eventually be less than QI. Let each of BD, DG be divided into equals, and though the cuts parallel to AD let lines be drawn and let EZ, HK, LM be joined. They will, in fact, be parallel to BG. The center of weight of parallelogram MN is on US, the center of weight of KC on TU, and that of ZO on TD. Therefore, the center of weight of the magnitude composed from all of them is on straight-line SD. Let it be, in fact, R, and let RQ be joined and extended, and let GF be drawn parallel to AD. Triangle ADG, in fact, has this ratio to all the triangles inscribed up from AM, MK, KZ, ZG similar to ADG, that which GA has to AM, since AM, MK, ZG, KZ are equal. And since triangle ADB also has the same ratio to all the similar triangles inscribed up from AL, LH, HE, EB, which BA has to AL, therefore triangle ABG has this ratio to all the mentioned triangles, that which GA has to AM.

Thankfully, you won't have to do anything like that in this book. But let's take it up a notch and give you more challenging shapes to work with. What about very irregular shapes? Is there an easy way to find the balance point for any shape?

ACTIVITY 1.3: Find the balance point of some U.S. states

You will need: *copies of the state outlines provided at the end of this chapter (either printed onto heavy card stock or glued to thin cardboard), a pencil, scissors, a pin, a piece of thread, and a small nut or washer (or any small, heavy object with a hole)*
NOTE: Once again, make sure you don't prick a hole in these shapes using your toothpick! The shape must balance on its own.

1) Let's start with Utah. Just for minute, pretend that the extra bump at the top is not there and put a pencil dot at the center of the main rectangle. Place that dot on the point of the balance stand and see what happens. Now move Utah around until it balances. Put a dot on the correct balance point. Compare the two dots. How does that extra bump affect the balance point? Is the correct point closer to the bump or farther away from it?
2) Now try Nevada. Using what you learned from Utah, take a guess where the balance point might be and mark it with a dot. Then balance Utah and mark the correct balance point. How close were you?



3) Next, we'll try Texas, but for this one we'll show you a sneaky trick for finding the balance point. Cut a piece of thread (about 20 cm) and tie a nut or washer onto one end. At the other end, tie a small loop. Put the pin through the small loop, then prick the pin into one corner of Texas (any corner is fine), as close to the edge as you can possible get it. Wiggle the pin a bit so that Texas hangs freely, maybe even swinging just a bit. (If you have trouble with Texas wanting to fall off the pin, hold it against a wall, but very gently so that it still can swing.) When both Texas and the weight have come to rest, you will need to draw a line where the thread crosses Texas. You can do this easily by making a pencil mark right where the thread crosses the bottom edge, then using a ruler to connect that dot with the pin prick. Now move the pin to a different corner. Any corner will do. Repeat this process and you will get another line across Texas. Do this for at least two more corners so that have four or more lines. The intersection of these lines will be the center of mass for the shape of Texas. Place this center dot on your balancing stand and see if it balances. If it doesn't, you should only have to make a very small correction.
5) Try this method for the other states.
CHALLENGE: Where is the balance point for Florida? Will Florida balance on your stand?

Let's stop and ponder something really strange. If a cardboard shape weighs 3 grams, all three of those grams are pressing down on the tip of your balance stand. If your balance stand was also a scale, it would register a weight of 3 grams. Yet only one small point of that piece of cardboard is touching the toothpick. Most of the cardboard is NOT touching the toothpick. It's like all of the mass of the entire shape is concentrated right at the place that is resting on the toothpick. How can this be? What is happening underneath all the rest of that shape—the parts that are not touching the toothpick? Don't they weigh something? This is the first of many strange concepts we'll come across in this book. For mathematical purposes, we can indeed assume that ALL of the mass of that shape is present right at that tiny point, and simply ingore the rest of the shape. Very odd indeed. And speaking of odd, the next activity will give us another strange idea to ponder.

In the next activity we'll begin using the term **center of mass**. We've been using the term "balance point" very loosely. We need to tighten up our defintion so that we can evaluate an object where the balance point is not resting on the tip of our toothpick. Sometimes the term "center of gravity" is used in place of "center of mass." The two terms are interchangeable as long as we are on or near the earth. We'll leave the discussion of mass versus gravity for a future chapter.

ACTIVITY 1.4: Find the balance point of some letters

You will need: *copies of the letter page provided at the end of this chapter (either printed onto heavy card stock or glued to thin cardboard), a pencil, scissors, a pin, a piece of thread, and a small nut or washer (or any small, heavy object with a hole)*

1) Start with the letters F and J. See if you can balance them just by experimenting.
2) Now try the letter L. Does it balance? Mark a dot where the balance point is. Then snip off the portion marked "1" and try again. Now does it balance? Mark the balance point. Snip off the portion marked "2" and try again, marking the balance point. Finally, snip off "3." How does the balance point change? What direction does it go? How are you changing the center of mass by snipping off parts of the letter?
3) Next, try the "V." Does it balance? As with the "L," snip off the number "1" sections then try balancing again. Then snip off the "2" sections. Does it finally balance? Where do you think the center of mass was before you snipped off any sections?
4) Now for the letter "O." Can you balance it? Why not? Where is the center of gravity for the "O"? Now snip off section "1." Can you balance this letter "C"? Snip off the "2" sections. Does it balance now? Where would you estimate the center of mass is? Finally, snip off the "3" sections. Where is the center of mass?
5) Lastly, try the lower case "d." Before you balance it, try to predict where the center of mass will be. Will it balance? After you find, out, predict what will happen if you snip of the "1" section. How will the center of mass change? Then go ahead and try it.

We've discovered that sometimes the center of mass is actually outside an object. Strange, but true. Let's do some more experimenting with objects that have their center of mass outside of themselves. This concept is a key to performing amazing balancing tricks.

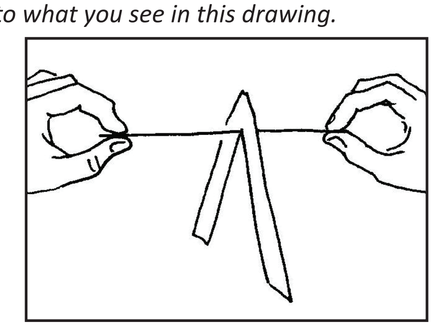
In this next activity, we will be balancing a cardboard V not on your balance stand, but on a tightrope made of thread. You will discover for yourself another basic principle of balancing.

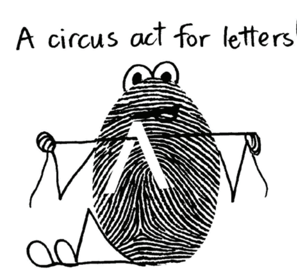
ACTIVITY 1.5: Balancing the letter V

You will need: *scissors, a piece of thread or thin string (20 cm is plenty), and a cardboard letter V with very long "arms."*

1) Cut a cardboard V that has very long "arms." We will be trimming these arms bit by bit, so make sure they are pretty long to start with. The exact length or width is not important. Just cut something similar to what you see in this drawing.
2) Have an assistant hold the ends of the string and pull it tight. Then put the V upside down on the string, so that it balances. Rock it a bit and see how well it maintains its balance. Remember what you discovered about the letter V in the previous activity. Where would the center of mass be for this V?
3) Now trim a bit off each "arm" of the V. Make sure you trim the same amount off each side. Try balancing it again. Does it still balance?
4) Trim the V again, the same amount off both sides. Balance it again. Rock it back and forth a bit. Is it still stable?
5) Keep trimming off a bit from both sides and checking how it balances each time. What happens when the arms get very short? Keep trimming until you reach a point where it does not balance anymore.
6) Take this unstable V and balance it on your balance stand. Does it balance? Is the center of mass inside or outside this V?

7) Where do you think the center of mass was for your original long-armed V? What happens as the center of mass gets closer and closer to the point of the V?





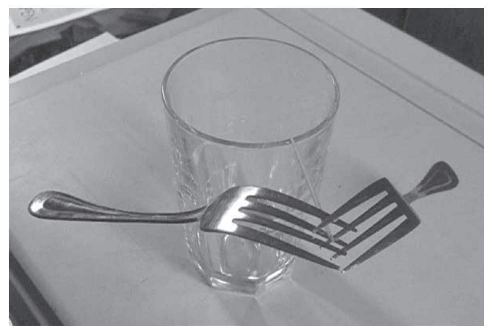
So now we've discovered that the way to achieve stable balance is to keep the center of mass below the balance point. (The string was the balance point in our last activity). Science museums in big cities sometimes have an exhibit where you can ride a bicycle on a tightrope, often very high up in the air. This can be done safely by making sure that there is a huge amount of weight well below the wire—so much weight that the bicycle can't possibly become unbalanced. This picture shows a famous "daredevil" act where a man got his wife to act as the center of mass below the rope. It's the same principle as the bicycles at science museums.

If you watch gymnasts do routines on the balance beam, you will notice that if they think they are starting to fall, they will quickly lower their bodies. They need to get their center of mass closer to the balance point in order to gain stability.

Now it's time for some tricks. Here are some balancing acts that use the principle of keeping the center of mass below the balance point.

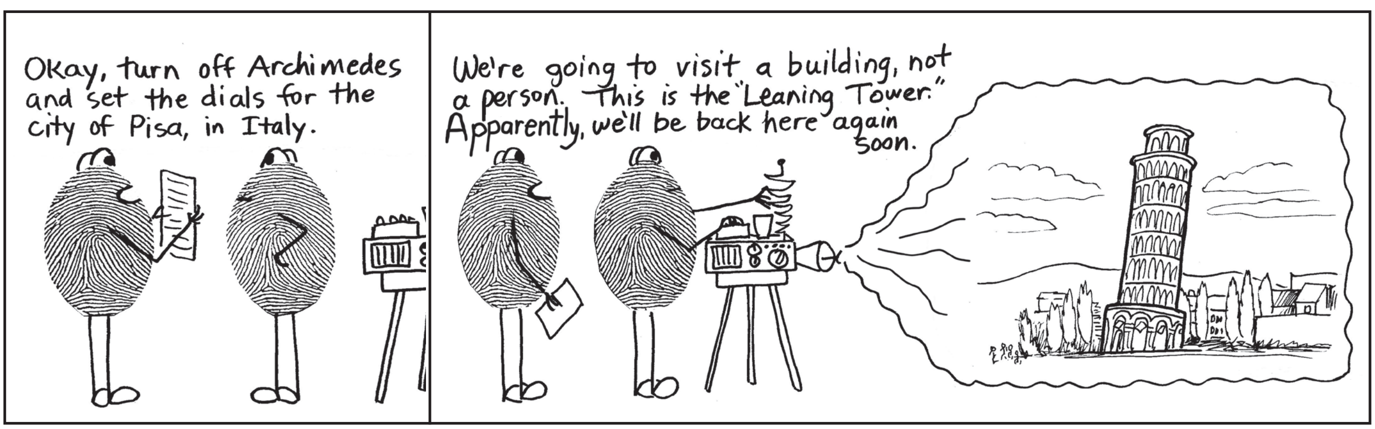
ACTIVITY 1.6: Balance two forks on a glass

You will need: *two forks, a toothpick, and a glass*
(Since these objects are often on tables at restaurants, this is a good trick to do while you are waiting for a meal!)



1) *Put the two forks together so that their tines interlock.*
2) *Wedge the toothpick into a crack where the forks intersect.*
3) *Rest the toothpick on the edge of the glass.*

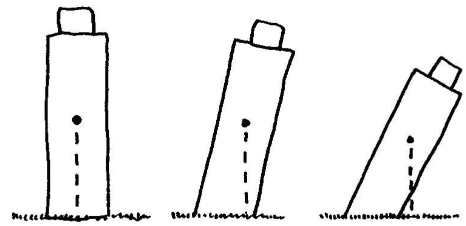
If you need a video demo, there is one on the playlist for this curriculum.
YouTube.com/TheBasementWorkshop, click on the "Discovering Motion" playlist.



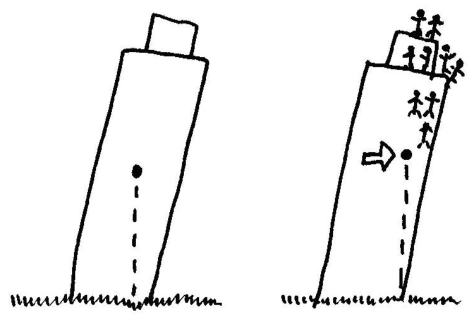
You have probably seen pictures of the Leaning Tower of Pisa. It didn't start out as a leaning tower, of course. It was built to be the bell tower of the cathedral in Pisa, Italy. Over the course of hundreds of years, one side has gradually sunk into the ground. What the builders didn't know was that the site they chose to build on had different types of rocks and soil underneath it. So basically, one part of the building was on harder rock and one part on softer rock. The softer rock started to compress and sink under the weight of the building, causing the tower to lean.

In the 1990s, it was determined that the Leaning Tower was leaning so much that is was no longer safe for people to be inside of it. Architects measured the rate at which the lean increased each year and calculated that eventually the tower would fall over. It would still be quite a few years until that happened, but they decided to go ahead and fix it now instead of waiting until the tower got any more dangerous. They figured out a way to adjust the foundation underneath the tower so that the higher side was brought down a bit. In 2004, the tower was declared to be safe. Tourists can now climb the stairs to the top and look out over the city.

How can you predict when an object is leaning enough that it will fall over? It's a matter of calculating where the center of mass is, then determining whether that center of mass is located over the base of the object. In this series of drawings, the dotted line shows where the center of mass is located over the base. In the first drawing, the center of mass is directly over the center of the base. In the third drawing, the building has leaned so much that the center of mass is no longer over any part of the base, but over the grass. When the center of mass of an object shifts so that it is no longer over any part of the base, the object falls over.

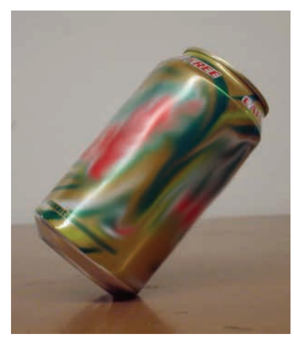


In these two drawings of the Leaning Tower, we've proposed a somewhat ridiculous scenario with a crowd of tourists grouped together on the lower side of the tower. So far, we've assumed that the weight of the tower is distributed equally, with no part of it being much lighter or heavier than any other part. However, when calculating center of mass, you have to take into account any areas that heavier (denser) than others. In our cartoon scenario, the weight of the tourists has greatly increased the weight of that side of the tower, causing a shift of the center of mass. With the additional weight of the tourists, the center of mass has moved closer to the edge of the base, and, therefore, closer to possible tragedy.

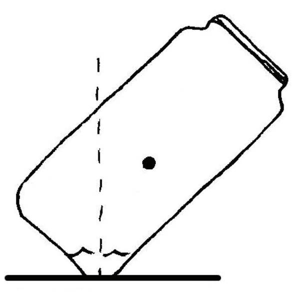


ACTIVITY 1.7: The leaning beverage can trick

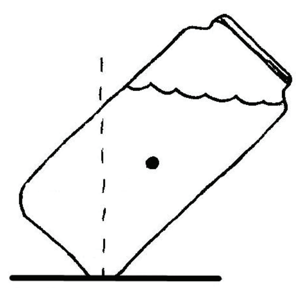
You will need: *an empty can of the type shown in this picture. The can must have a bottom that looks like this one, with an angular section between the side and the flat bottom. You'll also need some water that you can gradually pour into the can.*



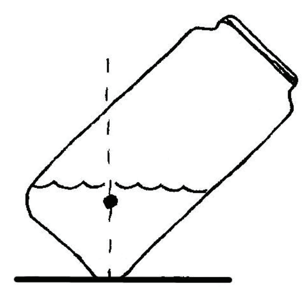
1) Start by trying to balance the empty can. (If you happen to have a full can, try to balance that as well. Prove to yourself that neither will balance.)
2) Pour water into the can until it is about 1/3 of the way full. Try to balance it again. If it does not balance, add or subtract a little bit of water until the can balances.
3) If you have balanced the can well, you will be able to push the can gently and get it to roll around in a circle!



If the can has too little water, the weight of the can itself will be the most significant factor in the center of mass, causing the center of mass to be far from the base. (The "base" is that very small edge touching the table.)

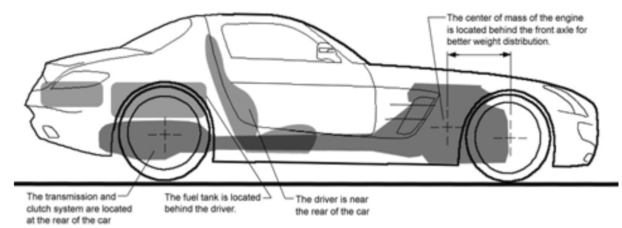


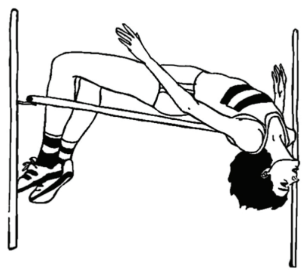
If the can has too much water, the center of mass will again be to one side of the balancing point, causing it to topple over.



If the can has just the right amount of water in it, the center of mass will be directly over the base.

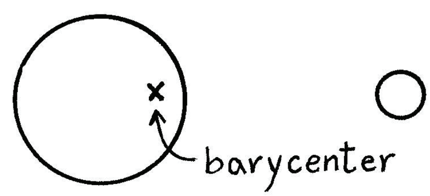
Center of mass is important in many branches of science and engineering. Engineers who design vehicles are very concerned about center of mass. Cars that have their center of mass closer to the ground are less likely to flip over (a special concern for racing cars that go around turns at very high speeds). If an airplane has its center of mass too far forward, it will tend to be less maneuverable and be difficult to control during take-off and landing. A plane with a center of mass too far back will be very maneuverable but will be less stable during flight. A helicopter must be designed so that the center of mass can shift backward when the pilot wants to go forward. This is why a helicopter looks like it is tilting nose-down when it is flying forward—the center of mass moves behind the rotor.





In sports, high-jumpers learn to bend in such a way that their bodies go over the bar even though their center of mass does not. Long jumpers shift their center of mass as efficiently as possible in order to maximize the power of their jump. Athletes shift their center of mass all the time as they run and jump, though they don't think about what they are doing in scientific terms. Scientists called ***kinesiologists*** *(kin-EE-see-OL-o-gists)* study body movement and develop equipment or therapies based on the science and math of body movement.

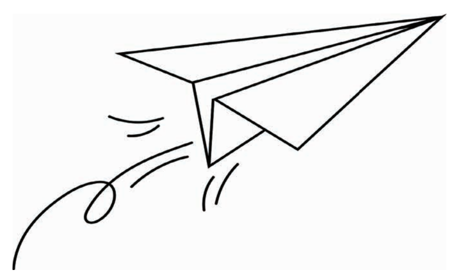
In astronomy, center of mass is very important. When a moon orbits a planet, or a planet orbits a star, they are actually both moving around a central point called the "barycenter." This point can be inside one of the bodies, or at a point in space. The Earth and the moon orbit each other at a point that is about 1,710 kilometers (1062 miles) below the surface of the Earth. It is this point, the barycenter, that orbits around the Sun. It's as if all the mass of both the earth and sun is concentrated into this one point.



ACTIVITY 1.8: Experiment with center of mass and airplane flight

You will need: *a piece of paper, a large paper clip, and instructions for making a paper airplane if you don't know how*

1) Make your airplane.
2) Put the paper clip on the nose. See how this flies.
3) Move the clip back a bit and try again. Better or worse?
4) Move the clip back again. Keep moving the clip back and see how it flies with the clip at the back. Is there an optimum place for the center of mass?
Extension: What happens if you put more than one clip on the optimum center of mass position?



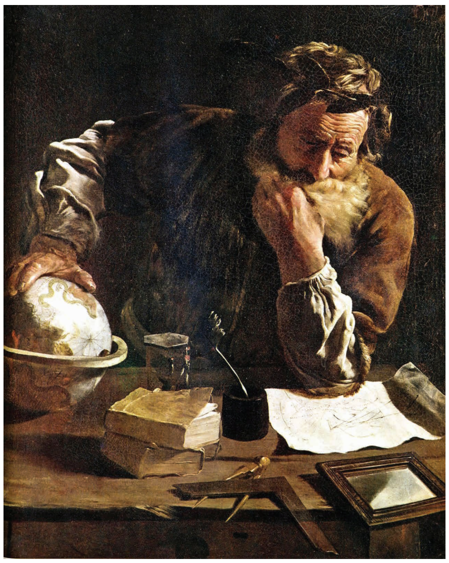
ACTIVITY 1.9: "The Impossible Hop"

You won't need anything special for this experiment, but some people like to place a dollar bill on the ground and make a bet with someone that they can't jump over it. You'll win the bet and keep the dollar bill most of the time.

1) Bend over and hold on to the tips of your shoes (or your toes if you are wearing just socks).
2) Try to jump forward without letting go of your shoes/toes. (In the dollar bill version, you place the dollar bill in front of their feet and tell them to jump over the dollar without letting go of their shoes/toes.)
3) Most people find this trick impossible because when you jump you shift your center of mass forward. In order to compensate for this sudden shift forward, you instinctive counterbalance using your arms. If you can't counterbalance with your arms because you are holding on to your toes, you will fall over.

ACTIVITY 1.10: Supplemental information about Archimedes

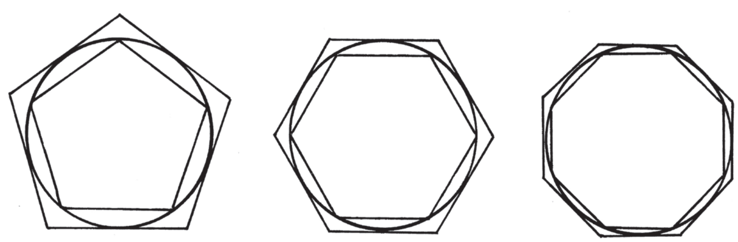
Archimedes was probably born in the year 287 BC. We are more sure of the year of his death than of his birth because his death occurred during part of a Roman war campaign that took place in 212 BC. As we learned in this chapter, he was born in the city of Syracuse, on the island of Sicily (shown in red on the map). His father, Phidias, was an astronomer, but that's all we know about his birth family. One of Archimedes' friends wrote a book about his life, but unfortunately, it has been lost or destroyed. We know the book existed because other ancient writers mentioned the book, we don't have the book itself. We also don't have any portraits of Archimedes, so we don't know what he looked like. Paintings of Archimedes, like the one shown below, are completely from the imagination of the painter.

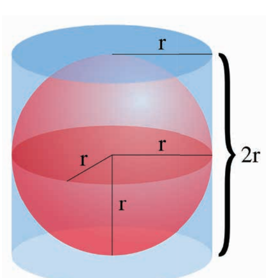


"Archimedes Thoughtful" by Domenico Fetti, painted in 1620

It is possible that as a young man, Archimedes traveled to Alexandria, Egypt, to complete his education. Alexandria had the best library in the world at that time and thus attracted scholars from all over the world. Wealthy families would often send their children to Alexandria in the same way that we send students off to universities. We know that Archimedes knew the head librarian, ***Eratosthenes*** *(air-uh-TOS-thuh-neez),* the man famous for estimating the circumference of the earth. (Yes, people back then knew that the earth was round, not flat.) One reason to think that Archimedes received part of his education in Alexandria is that he would have been about the right age to have been a student of the famous Greek mathematician ***Euclid*** *(YEW-klid*) who lived in Alexandria at that time. This would explain how Archimedes became such a mathematical genius at a relatively young age. Archimedes would also have studied philosophy in addition to mathematics. Socrates, Plato, and Aristotle pre-dated Archimedes, so he undoubtedly read their books (as handwritten scrolls, not as printed books).

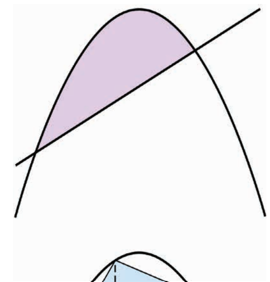
Mathematics in the ancient world consisted mostly of geometry. By Archimedes' day, mathematicians had already discovered everything we learn today in high school geometry, and much more. After learning everything known about geometry at that time, Archimedes went on to make many discoveries of his own. He was fascinated with the relationship of a circle's diameter to its circumference—the value that we call "pi" (3.1415...) He was able to approximate the value of pi (π) to be 3.141 by using straight-sided polygons both inside and outside of a circle. It is fairly easy to calculate the perimeter (distance around the outside) of a figure with square sides if you assume all the sides are the same length and you know the length of one side. In this figure, you can see that the measurement around the outside of the circle (its circumference) must be more than the perimeter of the smaller polygon, but less than the perimeter of the larger polygon. The correct value lies somewhere between the perimeters of these polygons. If you keep increasing the number of sides of the polygons, you will get closer and closer to the value for the circumference of the circle.

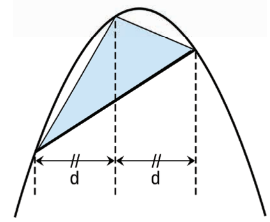


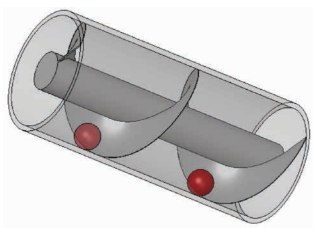


Archimedes considered that one of his greatest achievements was figuring out the relationship between the volume of a sphere and the volume of a cylinder that exactly contains the sphere, as shown in this diagram. The letter "r" represents the word "radius" which is the measurement from the center of a circle to its edge. The red sphere has exactly 2/3 the volume of the cylinder. It also has 2/3 the amount of surface area as the cylinder. The volume of the sphere is $4.3\pi r^3$. Why he considered this to be his greateest achievement is unknown. Calculating the value of "pi" would prove to be much more useful to future mathematicians and scientists.

Archimedes also studied parabolas. A ***parabola*** is the shape that a ball makes when you toss it in the air. The shape of the parabola will depend on the angle at which you toss it. Two parabolas are shown here. Archimedes used known rules of geometry to show that the area of the purple shape in the top drawing is equal to 4/3 of the area in the blue triangle in the bottom drawing. Not impressed? That's okay. Archimedes didn't need anyone to be impressed. He simply loved math. To him, it was a fun activity, not a chore.



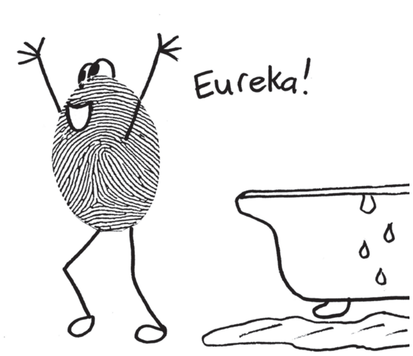




By Silberwolf (size changed by: Jahobr) - File:Archimedes-screw_one-screw-threads_with-ball_3D-view_animated.gif created by Silberwolf, CC BY-SA 2.5, https://commons.wikimedia.org/w/index.php?curid=2489813

Sometimes his geometry studies had obvious practial applications, and this allowed him to also become an inventor. His study of spirals inspired him to invent a tool we now call the "Archimedes screw," which consists of a long spiral shape inside a tube. (You can see a moving animation of this image by going to the Wikipedia page on Archimedes.) This device was a great help farmers who needed to water their crops. They could put one end of the tube into an irrigation canal and then crank the handle on the other end to bring that water up to higher ground. In modern times, you can find Archimedes screws in some types of machines, including grain elevators and combine harvesters used in agriculture.

By the time Archimedes was a young adult, his genius had caught the attention of the king of Syracuse. The most famous story about Archimedes comes from this period in his life. It is said that the king had commissioned the making of a gold crown and had given the goldsmith the amount of gold necessary. When he received the finished crown, the king thought there was something not quite right about it. He wondered whether the goldsmith had cheated him by using a cheaper metal to make the crown and then covered it with a thin layer of gold (keeping the rest of the gold for himself). He asked Archimedes to find a way to test the crown to see if it was make of pure gold, but without damaging the crown in any way. The story has several variations at this point. One version says that Archimedes was so obsessed with this problem that he forgot to do basic things like change his clothes or take a bath. When he began to stink, his friends took him to the public bath and tossed him in. Other versions simply have him stepping into a bathrub. All versions agree that the tub was so full of water that as he stepped in, some of the water sloshed out. He immediately realized that his body was now occupying the space that used to be taken up by that sloshed water. The volume of his body in the tube was equal to the volume of water displaced out of the tub. This principle (buoyancy) could be used to find the volume of the gold crown. When combined with information about the weights of silver and gold, he would be able to work a way to determine whether the crown was pure gold. He was so excited about his realization that he jumped out of the tub and ran down the street yelling **"Eureka!"** which was Greek for "I found it!" Being a bit absent minded, Archimedes was unaware that he was running down the street totally naked. We don't know how accurate this story is, but because it is so entertaining, there's no doubt it will continue to be told for many years to come.



The king of Syracuse would continue to rely on Archimedes for help, particularly with his military. Syracuse was constantly under threat of invasion by superior armies such as the Romans. Archimedes found ways to improve the design of his city's catapults so they could throw heaveir weights and be more accurate. He also designed a device called "the claw" which was said to be able to lift enemy ships out of the water, dumping all the sailors and their weapons into the sea. To defend their harbor, Archimedes found a way to use bronze shields as mirrors, angling them so they could direct beams of intense sunlight at ships, blinding the sailors or causing the sails to catch fire.

Archimedes wrote many short books, mostly on geometry or mathematical puzzles. In his book titled *The Sand Reckoner*, he tried to figure out how many sand grains would fill the universe. The most important thing we learn in this book is that Archimedes believed that the sun was the center of the solar system, not the earth. This idea somehow got lost for hundreds of years until it was rediscovered by Nicolaus Copernicus in the 1500s.

Some of Archimedes books were copied by scribes and preserved until modern times when they could be printed into bound books. Other books were have been lost or destroyed over the centuries. In 1906, an amazing discovery was made in the city of Istanbul (known in ancient times as Constaninople). A Danish professor was there to study a very old book from the 1300s which had been kept in a monastery library for hundreds of years. The pages were made of goat skin parchment, a common material for books at that time. Parchment was very expensive and was never wasted; scribes often recycled old parchment by carefully scraping the ink letters. The professor noticed some very light lettering behind the darker lettering of the Medieval prayers and wondered if this parchment had been recycled. You can see some of this light lettering, plus some circles, in on the pages shown here. After further examination, he realized that this light lettering was a copy of text by Archimedes. When experts were able to examine the entire book, they found three of Archimedes' books that no one had ever seen. What a discovery!

Recycled parchment books are called "palimpsests."

ACTIVITY 1.11: Supplemental Videos

There is a YouTube playlist for this book located at YouTube.com/TheBasementWorkshop. Click around until you find the playlist called “Discovering Motion.” (You might have to click on “Created Playlists.”)

I’ve already watched all these videos and made sure they are don’t contain anything you shouldn’t be watching, but even so, make sure your parents or guardians know you are accessing YouTube.

I used to be able to put labels on the videos marking which ones go with which chapter, but I can’t do that anymore, so I just try to make sure they come in approximately the right order as you read through the book. For this chapter, watch the first few videos that are about center of mass/gravity and Archimedes.

ACTIVITY 1.12: Review questions

See if you can remember the answers to these questions. If not, go back into the text and find the answer.
(There is an answer key in the teacher's guide.)

1) Archimedes was born in the city of ______________________ on the island of _______________.

2) TRUE or FALSE? The balance point of a wooden rod can also be called the center of gravity.

3) TRUE or FALSE? A trapezoid is a special type of parallelogram.

4) To find the center of a triangle, you find the __________________ of each side and then connect that to the angle directly opposite to it. The place where these three lines __________________ will be the center of gravity.

5) TRUE or FALSE? "Center of gravity" and "center of mass" are the same as long as we are on Earth.

6) TRUE or FALSE? Some shapes have their center of mass at a point that is outside the shape.

7) TRUE or FALSE? When doing calculations, you can assume that the entire mass of the object exists at a single point, the point we call center of mass. Mathematically, the rest of the object weighs nothing.

8) TRUE or FALSE? An balanced object becomes less stable if its center of mass is lowered.

9) TRUE or FALSE? The Leaning Tower of Pisa was designed to lean, in order to attract tourists.

10) What was the original function of the Leaning Tower? ______________________________

11-13) Name three practical applications for center of mass in industry or sports:

____

____

____

14) TRUE or FALSE? A barycenter is the point around which two objects orbit.

15) Who was Eratosthenes? _________________________________

16) What scientific achievement is Eratosthenes famous for? ____

17) Was Archimedes older than Euclid? _______

18) Was Plato older than Archimedes? ____

19) The measurement of the outside (perimeter) of a circle is called its _______________________.

20) The distance from the center of a circle to its edge is called the ________________.

21) TRUE or FALSE? The Archimedes screw can only be used to pump water.

22) TRUE or FALSE? No one knew that the earth goes around the sun until Copernicus in the 1500s.

23) TRUE or FALSE? Archimedes was against war and refused to use his genius for anything related to military.

24) Archimedes wrote a short book in which he tried to figure out how many ____________________ it would take to fill the universe.

25) Why were parchment books sometimes recycled? ____

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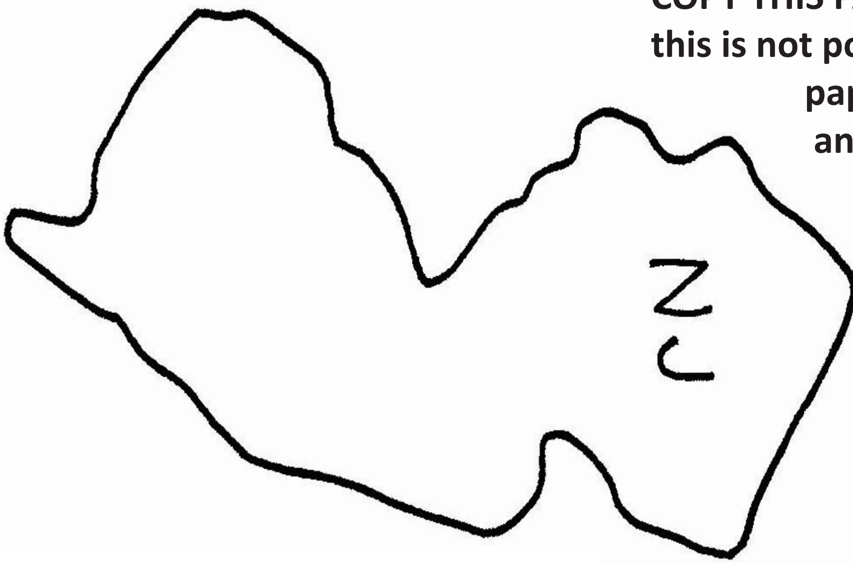
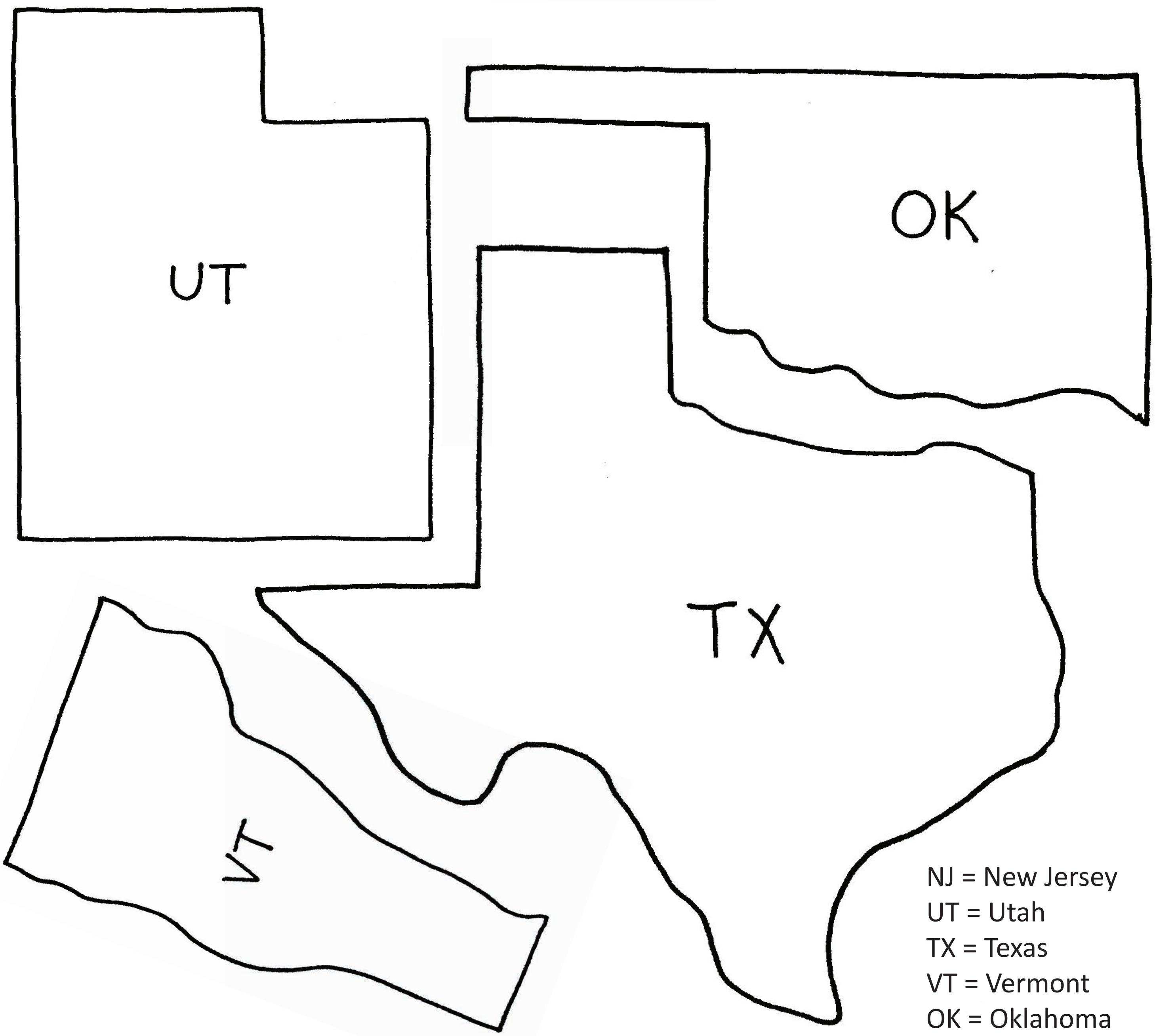
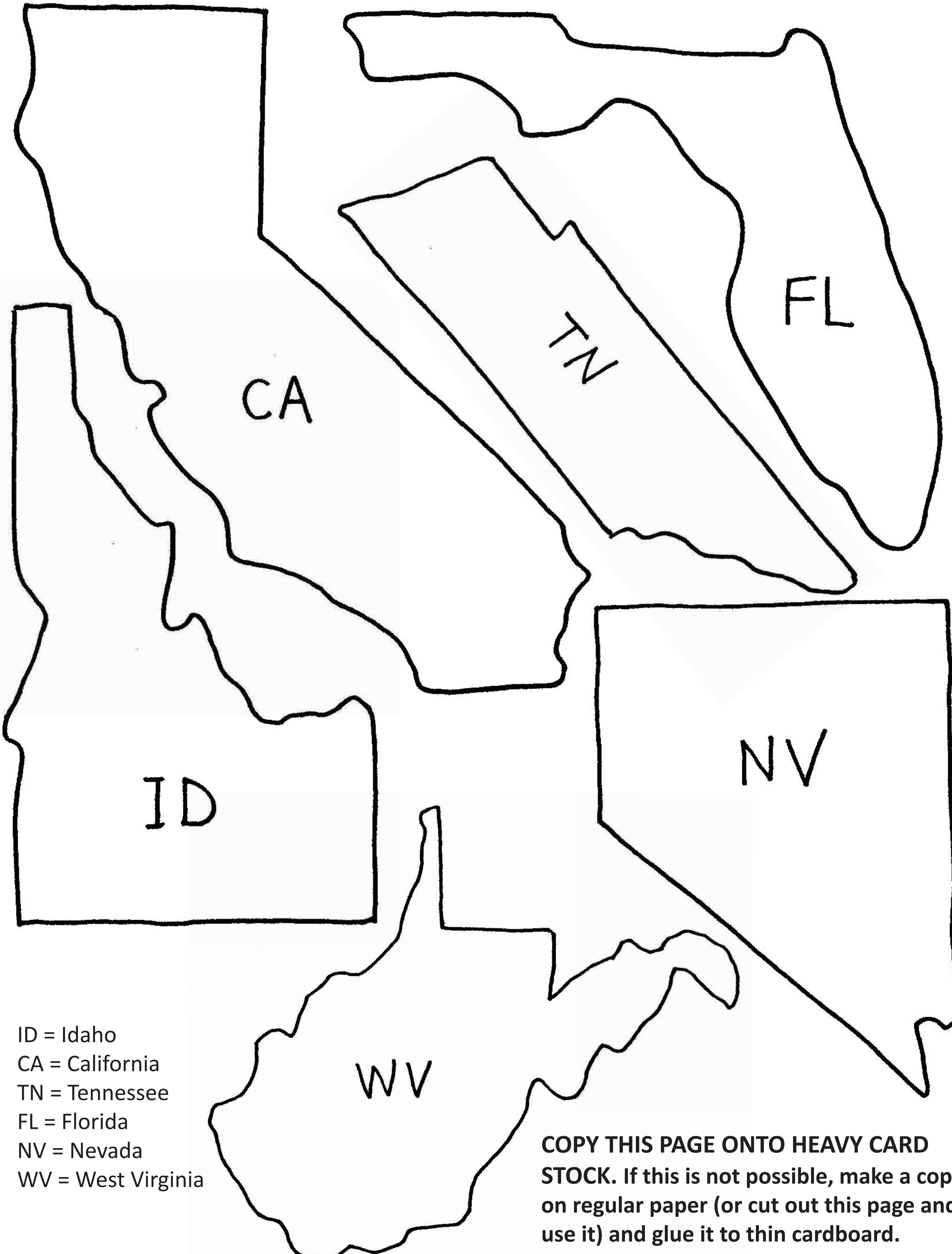


NOTE: These states are not drawn to scale. Vermont and New Jersey are much smaller than Texas and Oklahoma! In this activity, we are interested in just their shapes, not in their relative sizes. We could have used other odd shapes, but state shapes are fun to work with.



NJ = New Jersey
UT = Utah
TX = Texas
VT = Vermont
OK = Oklahoma



ID = Idaho
CA = California
TN = Tennessee
FL = Florida
NV = Nevada
WV = West Virginia

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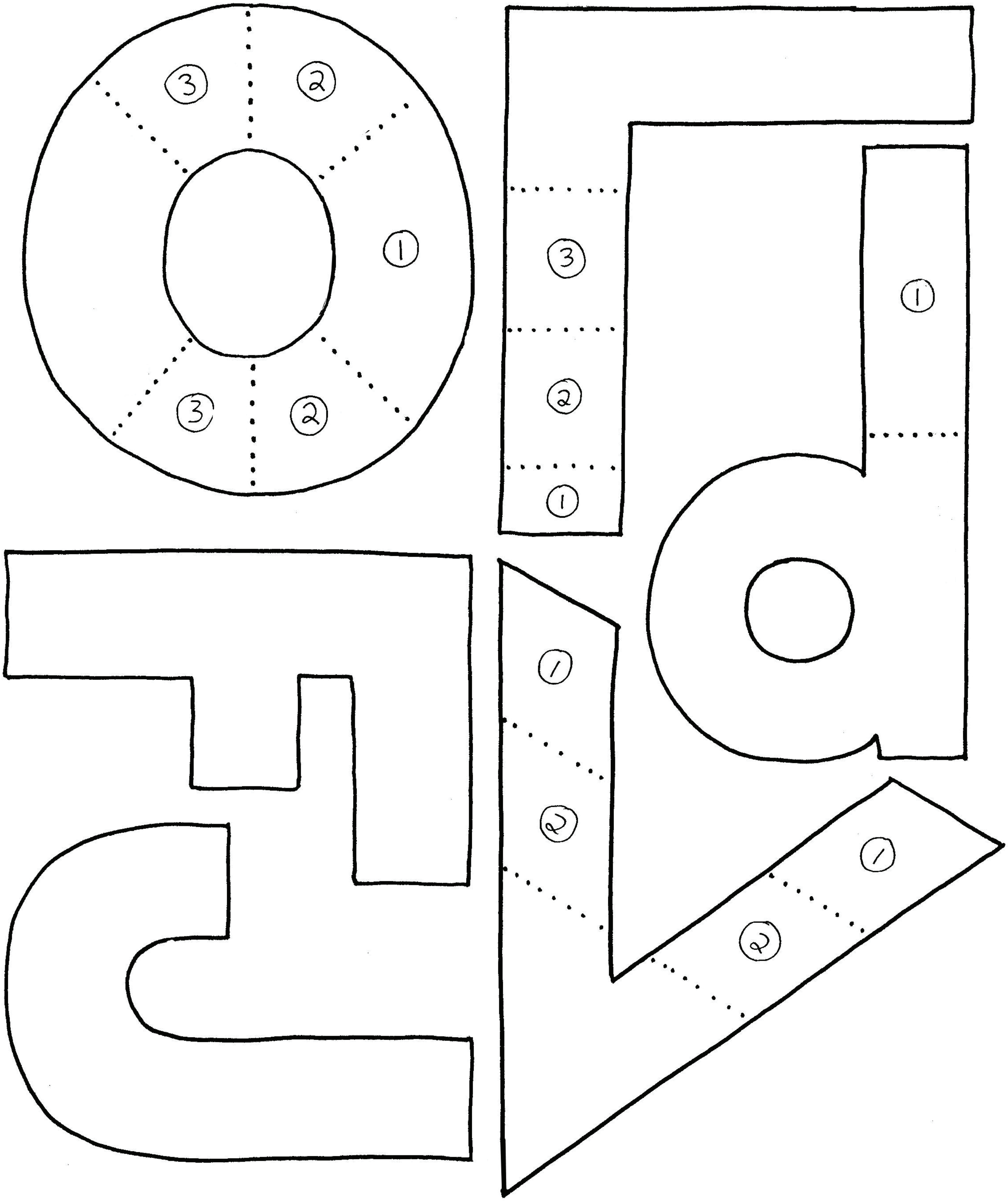


Copy this page onto heavy card stock paper. If this is not possible, you can copy it onto regular paper (or cut this page out of the book) and glue to thin cardboard.)

SUPPLEMENTAL ACTIVITES

NOTE: The numbering of these extra activites picks up where the numbering stopped in the chapters. This will avoid confusion between chapter activities and supplemental activiities.

CHAPTER 1:

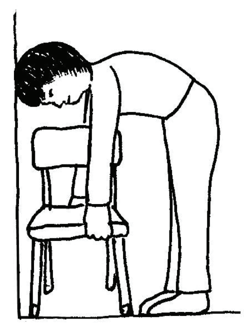
ACTIVITY 1.13: Immobilize someone with your pinky finger

You will need: a chair and a volunteer for this activity.

1) Tell your volunteer to sit in the chair, with his/her back against the back of the chair and hands in lap.
2) Place your pinky finger on his/her forehead. (You can use any finger-- it does not have to be the pinky.)
3) Tell your volunteer to stand up. Use your finger to keep his/her head from tilting forward.
4) Your volunteer will probably not be able to stand up.

When you are standing, your center of mass (somewhere in your abdomen) is directly over your feet. When you are seated, your center of mass is above the seat of the chair, not over your feet. In order to stand up you need to move your center of mass from over the chair to over your feet. To accomplish this, you need to lean forward. Strangely enough, the amount of force needed to keep someone from leaning forward is not all that much. You can exert this force with one finger.

ACTIVITY 1.14: The "girls always win" chair lifting challenge



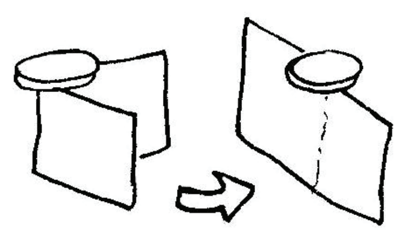
You will need: two volunteers (one male and one female), a chair, and a wall against which you can set the chair. The older the volunteers are, the better. The difference between the center of mass for boys and girls (under the age of 12 or so) might not be enough to make this trick work.

1) Place the chair against the wall.
2) Have one volunteer bend over the chair so that his/her head touches the wall and his/her upper body is parallel to the floor, as shown in the diagram. Tell the volunteer to try to lift the chair and then stand up.

Men and women have their center of mass in different places. Men tend to have broad shoulders and narrow waists, giving them a higher center of mass. In this case, a higher center of mass is a disadvantage because adding the weight of the chair makes the center of mass even higher, creating an impossible situation where there just isn't enough counterbalancing weight in the lower half of the body. Girls and women have their center of mass closer to their hips, which in this case is an advantage because even with the added weight of the chair, the overall center of mass is close enough to being over the feet that the body is able to right itself.

(NOTE: In my classroom, we found it hard to get the "correct" results, probably due to the young age of the boys. Our boys could lift the chair. For young students, try a relatively small chair with a book or two sitting on it.)

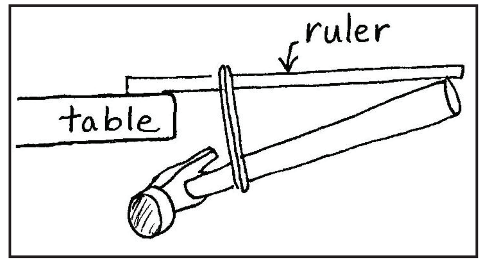
ACTIVITY 1.15: Balancing a coin on a dollar bill



Did you know you can balance a coin on a paper bill? The trick is to fold the dollar a bit, rest the coin on the folded corner, then slowly open the bill. If your hands are steady and you pull the ends of the bill very slowly, the coin will balance right on the edge! If you want to see a video demonstration of how to do the trick, go to YouTube.com/TheBasementWorkshop and find the "Discovering Motion" playlist.

ACTIVITY 1.16: Balance a ruler with a hammer

You will need: a hammer, a one-foot ruler (30 cm), and some string or a strong rubber band



This balancing act looks like it should not be possible. The hammer hangs precariously underneath the ruler, looking like it should make the ruler fall off the table. But there it hangs, perfectly balanced!

1) Take the rubber band or string and make a loose loop around the hammer and ruler, as shown in the picture.
2) Make sure the end of the hammer is touching the ruler, and then position the ruler at the edge of a table, as shown. (You might have to reposition the string/rubber band a few times to get it just right.)
3) Why does this trick work? Analyze where the center of mass might be. Where is the balance point? What is the heaviest part of a hammer?

ACTIVITY 1.17: Make a "balancing bird"

For each bird, you will need: a copy of the following pattern page printed onto heavy card stock, scissors, tape, two pennies, a toothpick and a glue stick (Optional: colored pencils or markers to color the bird)

Follow the directions on the pattern page.

Answers to Activity 1.12 Review questions:

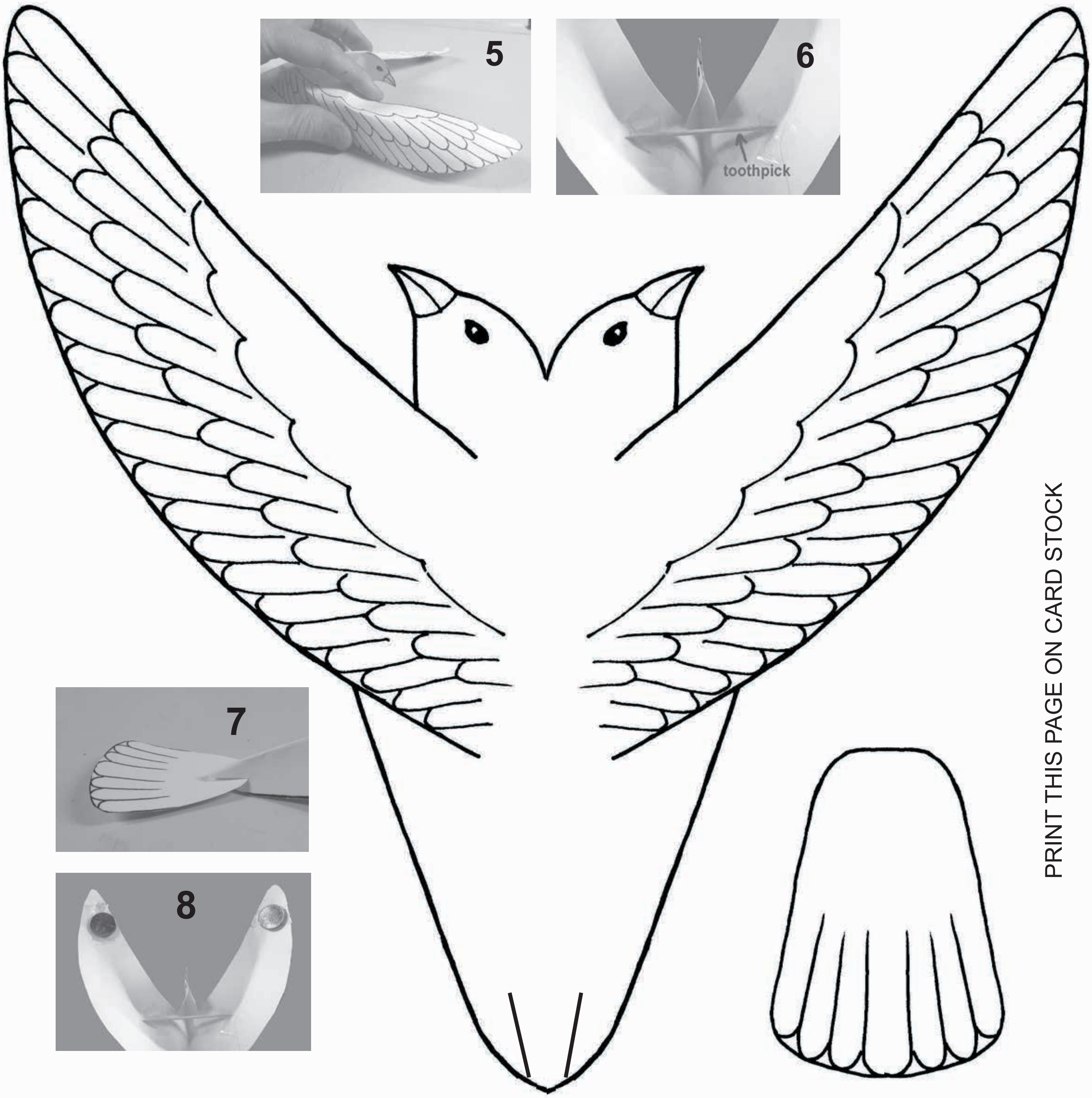
1) Syracuse, Siciliy 2) T 3) F 4) mid-point or center. intersect
5) T 6) T 7) T 8) F
9) F 10) bell tower
11-13)
car-- keep the center of mass low to make it stable around turns
plane-- center of mass affects how it files
athletes-- shift their mass to make efficient jumps
14) T 15) librarian of Alexandria 16) estimating the circumference of the earth
17) no 18) yes 19) circumfernece
20) radius 21) F 22) F
23) F 24) grains of sand25) parchment was very expensive.

BALANCING BIRD TOY

You will need:

• two pennies, a toothpick, clear tape, glue stick (optional: coloring supplies)

1) Do any coloring you want to do. 2) Cut out bird and tail. Make sure to cut along the wing lines that go into the body area. 3) Fold the bird in half. 4) Apply glue stick to inside of the forward half of head (eye and beak area) and stick halves together. (Note: Beak can be reinforced with clear tape if it seems too flimsy.) 5) Make a slight crease along the lengths of the wings, to stiffen them. 6) Tape toothpick to underside of wings, across the center, (like the cross bar of a kite). 7) Insert tail piece and secure with tape on the underside. 8) Roll two pieces of tape and apply one to each penny. Stick pennies on the undersides of the ends of the wings and then check balance. Adjust the pennies if necessary to make the bird balance well. Once pennies are in the right place, secure them with a little more tape.



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