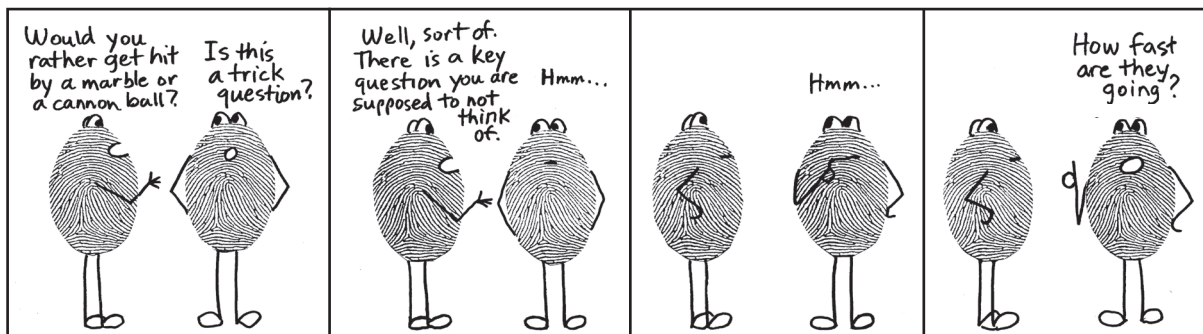


Adventure 7: Calculating and colliding with Newton (Newton's 2nd and 3rd Laws)



You guessed it! There is more to force than just mass. A cannonball has a lot more mass than a marble, but if you got hit with a cannonball that was only going at a speed of 1 km/hour (slower than the speed of walking), you would easily be able to catch it and not get hurt. On the other hand, if a marble came at you as fast as a bullet, it could cause some serious harm to your body.

Newton also knew this to be true. Since childhood, he had watched things fall and knew that an apple dropped from knee-height would hardly be bruised, but apple dropped from the top of a roof would smash and splatter as it hit the ground. He knew that the longer something fell, the more acceleration it would experience. Every second an object falls, it picks up additional 9.8 meters per second. He figured out that the relationship between force, mass and acceleration is actually a fairly simple one. He addressed the relationship between force and mass in his famous Second Law. Unfortunately, the way he wrote this second law is a bit harder to understand than his first and third laws. Here is how he stated his **Second Law**:

LEX II: The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

He went on to say, "If any force generates a motion, a double force will generate double the motion, a triple force, triple the motion, whether that force be impressed altogether and at once, or gradually and successively." This statement, along with other things Newton said, has led scientists to a mathematical way to express this second law. There is general agreement that the best way to summarize **Newton's Second Law** is like this:

$$\text{FORCE} = (\text{Mass})(\text{Acceleration})$$

$$F=ma$$

Remember, when two letters or numbers are next to each other (with or without parenthesis) this means they are being multiplied.

Force equals mass times acceleration. This is how you see it stated in many textbooks. This accomplishes two important purposes. First, it makes the law much easier to remember, and second, it gives it an easy and obvious application. You can use this simple formula to calculate force.

But first, let's review how we measure force. The unit of force is the **newton, N**. One newton is the force it takes to give 1 kilogram of mass an acceleration of 1 meter per second per second. A pineapple has about 1 kg of mass. The force it takes to move a pineapple (across a frictionless surface) the distance of one meter in one second is 1 newton of force. If that 1 N of force kept going after the first second, the pineapple would move 2 meters in the following second and 3 meters in the third second.

Gravity gives an acceleration of 9.8 m/sec², so 1 N (only 1 m/sec²) is not much force compared to gravity. Even if we just hold the pineapple and don't drop it, the pineapple is still experiencing the acceleration due to gravity. That's one of the trickiest parts of understanding gravity. We still call it "acceleration due to gravity" even if the object is resting on a surface, not falling. The formula F=ma tells us that we can multiply the mass of an object times the acceleration due to gravity and get the downward force of the object in newtons. For the pineapple, 1kg x 9.8/sec²= 9.8 newtons. When you hold a pineapple, you are feeling a force of 9.8 newtons pushing down on your hands.



Our formula, $F=ma$, tells us that Force is proportional to mass. If we keep the acceleration at 1 m/sec^2 , this will give us some very easy math because. If $a=1$, then we have just $F=m$. (Multiplying a number by 1 doesn't change the number, so "m" times 1 is just "m." This means we can think of "a" as not being there.) " $F=m$ " tells us that we need 2 newtons of force to accelerate a 2 kg weight, 3 newtons of force to accelerate a 3 kg weight, and so on. Any increase in mass will be directly proportional to the amount of force needed.

Another thing we can notice about $F=ma$ is that there are many ways to make the same amount of force. If we set the units aside and just look at numbers, and we let force equal 24 N, we can see that as mass increases, acceleration will decrease, and vice versa. $F=24$ $24=(6)(4)$ $24=(4)(6)$ $24=(8)(3)$ $24=(3)(8)$ $24=(12)(2)$ $24=(2)(12)$ If mass is large, acceleration will be small. If acceleration is large, mass will be small.

$F=ma$ works in zero gravity (free fall), too. Scientists on the International Space Station set up a way to demonstrate this. They used a bungee cord strung across the cabin to make sort of a "sling shot" that could provide force. They kept the force the same by always pulling back the cord the same amount. Then they launched objects of different masses and watched the difference in acceleration. In fact, why don't you watch this right now?



ACTIVITY 7.1: Watch the ISS demonstration

Use the Youtube playlist, or search for "STEMonstrations: Newton's 2nd Law of Motion"

ACTIVITY 7.2: Do your own demonstration of $F=ma$

You will need: a large plastic cup, scissors, two meter sticks, a stack of books, a sheet of card stock, tape, colored pencils, balls of different sizes and masses (ex: ping pong ball, golf ball, "super-ball," marble, large marble, gum ball, ball bearing, etc.)

NOTE: If you constructed a ramp for the Galileo's ramp experiment, you can use this ramp instead of making another one.

How to set up:

- 1) Turn one of your meter sticks into a ramp. Cut thin strips of card stock paper and tape them along the sides to make "railings" so that the balls won't jump off the track. 2) Cut your plastic cup as shown in the diagram on the next page.
- 3) Place the ramp on a stack of books, and position the other meter stick right at the end of the ramp. (see diagram)

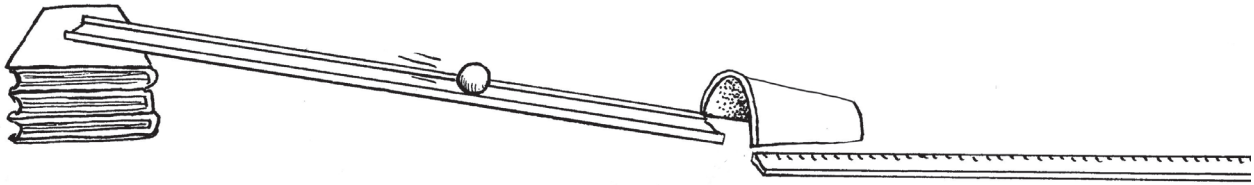
How to run the experiments:

We won't be able to put all of our data onto one graph, so we'll do separate experiments for mass and acceleration. First, we'll keep mass the same for all the trials and just vary acceleration by adjusting both the height of the ramp and where we place the ball on the ramp. Then we'll switch and keep acceleration constant while we vary the mass. (Ideally, you would measure the mass of each ball you are using, so you can quantify mass. However, if you don't have a scale that measures in grams, you can just list the balls in order, from lightest to heaviest.)

In every case, the final result will be a measure of Force. Trying to measure force in newtons will be too difficult in this experiment, so we're going to use our own custom-designed measurement. We're going to determine force by letting the ball hit the back of the paper cup, which will cause the cup to slide forward a bit. We'll measure how many centimeters the cup moves from its starting point.

Varying Acceleration

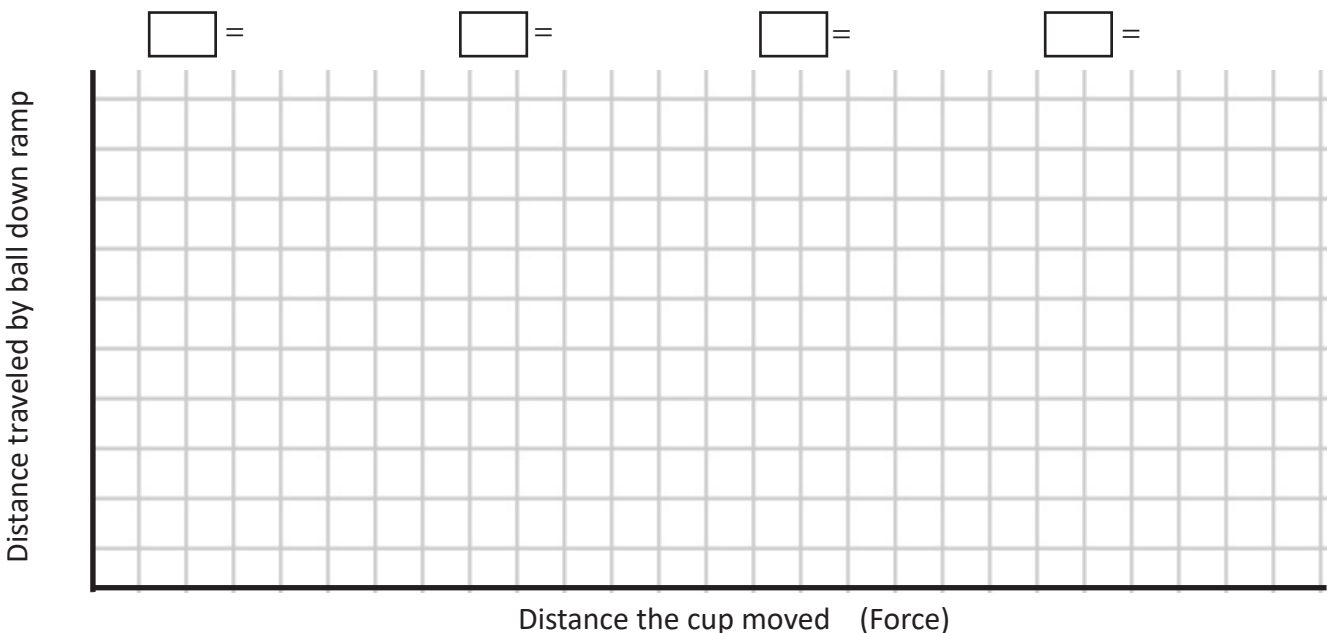
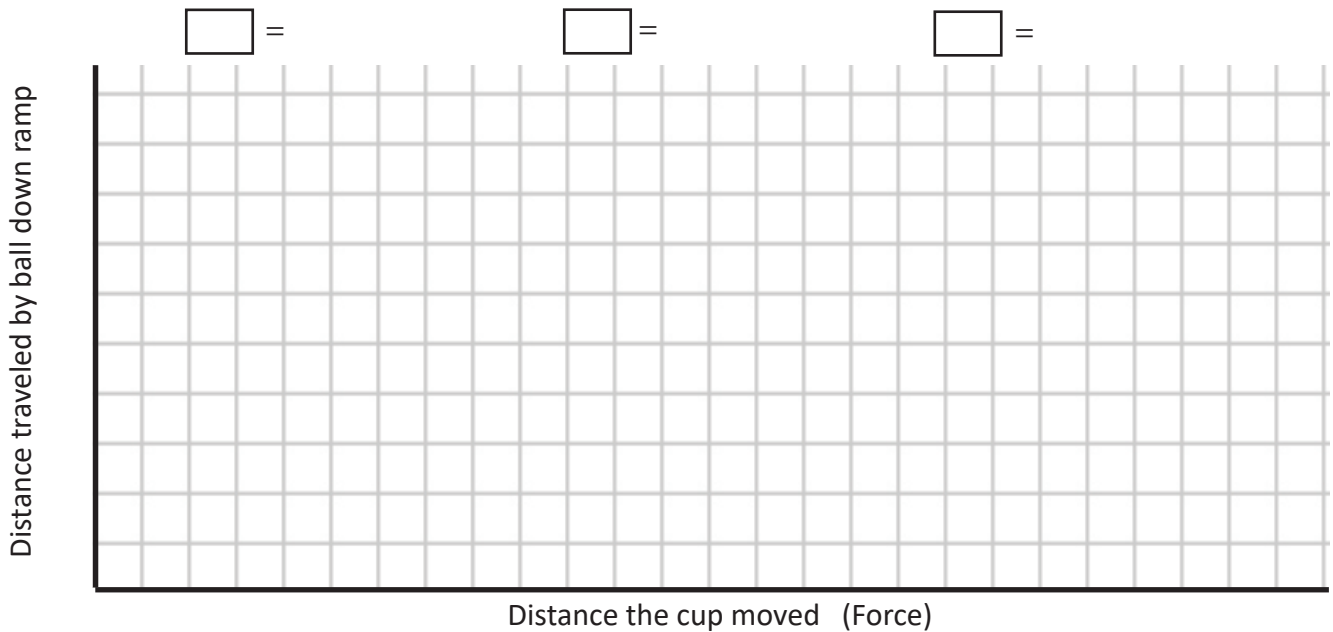
- 1) Try out all the balls at various heights of the ramp, and choose a ball that will give good results, pushing the cup every time even if the ramp is low, but not pushing it too far if the ramp is high.
- 2) Write the units on the first graph. The bottom scale will likely be 2 centimeters per block, but you might have to adjust it after you see how far the cup will go. Ideally, your data will span most of the length of the graph. The vertical scale on the left side will be in centimeters, using 10 cm per block.
- 3) Choose one of your colored pencils to represent the ramp at its lowest point. Color in the first square above the graph and write "low" after the equal sign (or write the actual height of the stack of books).



- 4) Put the ball at the top of the ramp and let it go. Then check to see how far the plastic cup was displaced. Record on graph.
- 5) Replace the cup so it is again flush with the start of the meter stick. Now place the ball at the 75 cm mark and let it go. Record displacement of cup. Then place ball at 50 cm mark, then another run at the 25 cm mark. Record result each time.
- 5) Then add more books to the stack to make a medium-height ramp. Change to new colored pencil, fill in second rectangle.
- 6) Run balls again at full ramp, 75 cm, 50 cm, and 25 cm. Record data on graph.
- 7) Make the ramp tall by adding more books. Choose another color, fill last rectangle and write "high." Do the four runs of the ball again and record data. / You might want to connect the dots of each color, to make colored lines or curves.

Varying Mass

- 1) Set the ramp at medium height. Assign colors to each of your balls.
- 2) Run each ball at full ramp, 75 cm, 50 cm, and 25 cm. Record the data for each ball with the correct color.
- 3) Connect dots of same color to make colored lines or curves. / NOTE: Samples of student work are given in teacher's section.



Questions for Activity 7.2: (discussion and suggested answers in teacher's section)

- 1) Did the more massive balls produce more force?
- 2) Did acceleration produce more force?
- 3) Which had more effect on force—the height of the ramp or the position of the ball on the ramp?
- 4) Were the shapes of the lines/curves similar for both graphs? Why this result?
- 5) Did your experiments confirm Newton's assertion that both mass and acceleration affect force?
- 6) Were there any instances where a small amount of acceleration and a large mass gave about the same force as a large amount of acceleration and a small mass?

We can use the formula $F=ma$ to solve problems that ask you to find any of the variables: force, mass or acceleration. If you know two of them, you can find the third.

For example, let's consider a mass of 5 kg and an acceleration of 10 m/sec². How much force will be needed to accelerate a 5 kg watermelon at the rate of 10 m/sec²?

$$F = (5 \text{ kg})(10 \text{ m/sec}^2) = 50 \text{ newtons.}$$

Using newtons as our measurement of force lets us avoid writing out "kg(m/sec²)" because newtons are, by definition, in kg(m/sec²).

5 kilograms is about 10 pounds, which is pretty big for a watermelon, but some do grow to this size. You may have noticed that we used 10 m/sec², which is approximately the same as the acceleration due to gravity. If someone on the space station launched a 5 kg watermelon with an acceleration of 10 m/sec², it would hit the wall with about the same force that it would if it had been dropped on earth.

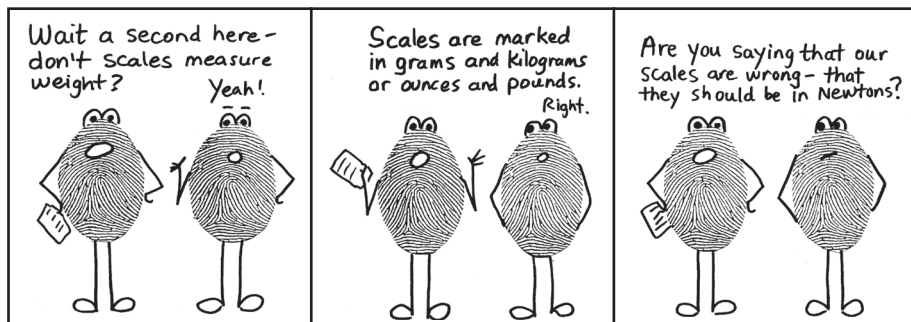


We can also use the formula to find either mass or acceleration. For example, if we know the force being applied is 100 newtons, and we know the acceleration is 5 m/sec², what is the mass of the object? We set it up like this: $100 = (m)(5)$. What is "m"? We know that 20 times 5 is 100, so "m" must be 20 kg.

Or, if we know the force and the mass, we can find acceleration. If the force is 24 newtons, and the mass of the object is 8 kg, how much acceleration will this amount of force give to this object? $24 = (8)(a)$. We know that 8 times 3 is 24, so the acceleration must be 3 m/sec².

Another use for $F=ma$ is to calculate weight using newtons instead of kilograms. You'll remember that your spring scale was labeled in both grams and newtons. Calculating your weight in newtons is very easy if you use $F=ma$. You can measure your mass in kilograms (an inertial balance would be ideal, but a regular scale can be used), and you know that the acceleration due to gravity is 9.8 m/sec². Just multiply your mass by 9.8 and your answer will be in newtons. If someone has a mass of 100 kilograms, their weight would be 980 newtons.

How much is 1 N? A candy bar with a mass of 102 g (the candy shown here has "102 grams" marked on the wrapper) has a weight of almost exactly 1 N ($.102 \times 9.8$).



Yep, that's what we're saying. Ideally, we'd all measure our *mass* with inertial balances, but this is not practical, so we use scales that make use of gravity. But because there isn't really any difference between mass and weight while you are on planet Earth, we can fudge a little bit and mark scales with kilograms or pounds. But to be technically accurate, our bathroom scales should be marked in newtons.

ACTIVITY 7.3: A few simple math problems with F=ma

- 1) The worldwide average weight of a human is 62 kg. How many newtons does the average human weigh? _____
- 2) How much force will be needed to accelerate a 20 kg cannonball at 30 m/sec²? _____
- 3) If someone's weight is 490 N, what is their mass in kilograms? _____
- 4) A trebuchet launches a large 15 kg pumpkin with a force of 135 newtons. What is the acceleration? _____
- 5) A lab mouse tips the scale at 8 grams. What is its weight in newtons? _____ (Remember to convert g to kg.)
- 6) An astronaut in the ISS pushes a 200 gram wrench toward the wall. It hits the wall in exactly 2 seconds and the wall is 10 meters away from the astronaut. The wrench hits the wall with about how many newtons of force? _____
- 7) How much force is need to accelerate a 1,700 kg car at 3 m/sec²? _____
- 8) A very bad dog jumped up on the kitchen table. The dog is a large breed—his mass is 60 kg. How much force is the table experiencing because of the dog? _____

Even with simple problems like these, you always have to read carefully and pay attention to the units being used. We did not throw anything tricky at you (only converting grams to kilograms) but a teacher might give you a problem in miles per hour, and you will have to convert it to meters per second. Or if a weight is given in pounds or grams, you will have to convert to kilograms. **The answer MUST be in kilograms, meters, and seconds** (which are the units involved in newtons).

Another piece of trickery that teachers might throw at you is making you calculate acceleration instead of giving it to you. For example, they might tell you that the box started out at rest, but after 4 seconds it is now moving at the speed of 2 meters per second. We don't know the velocity at the 1-second mark, or the 2-second mark, or the 3-second mark—only the 4-second mark. So we just divide the final velocity, 2 m/sec, by the number of seconds it took to reach that velocity, 4 seconds. $2/4 = .5$ So the acceleration was an average of .5 meters per second for each of those 4 seconds. We don't need to know all the details about what happened at each second, just those final numbers. The mathematical way to write this is: $a = v/t$

$$\text{ACCELERATION} = \frac{\text{Velocity}}{\text{Time}}$$

Acceleration equals velocity divided by time

Here are some more problems that are based on F=ma, but you might have to do some unit conversions or figure out acceleration. Always convert to kg, m, and seconds.

ACTIVITY 7.4: More math problems with F=ma

$1 \text{ pound} = 16 \text{ ounces} = .45 \text{ kg}$ $1 \text{ mile} = 1.6 \text{ km}$

- 1) On the ISS, astronauts rigged up a sling shot that gives exactly 1 N of force every time. If they put a 1-pound object into the sling shot, what acceleration can they expect to see? _____
- 2) A nerf gun can shoot darts with a force of 2 N. If a dart weighs 3 ounces, how much acceleration will the dart have as it leaves the gun? _____
- 3) An airplane started out at rest on a runway, and at the end of 10 seconds it was going 200 km per hour. If the mass of the airplane is 250,000 kg, how many newtons of force are the engines producing? _____ ($a=v/t$)
- 4) An 8-ounce ice hockey puck slides across a friction-free ice surface. A 2.7 N force is being continuously applied to the puck. What will the acceleration of the puck be? _____
- 5) In the frictionless environment of outer space, a space station launches a small probe with a force of 10 N. After 50 seconds, the probe's speed is measured at .25 km per second. What is the mass of the probe? _____ (Use $a=v/t$ to find a. Don't forget to convert km to m!)



ACTIVITY 7.5: $F=ma$ with skateboards (This activity also gives you a preview of Newton's third law.)

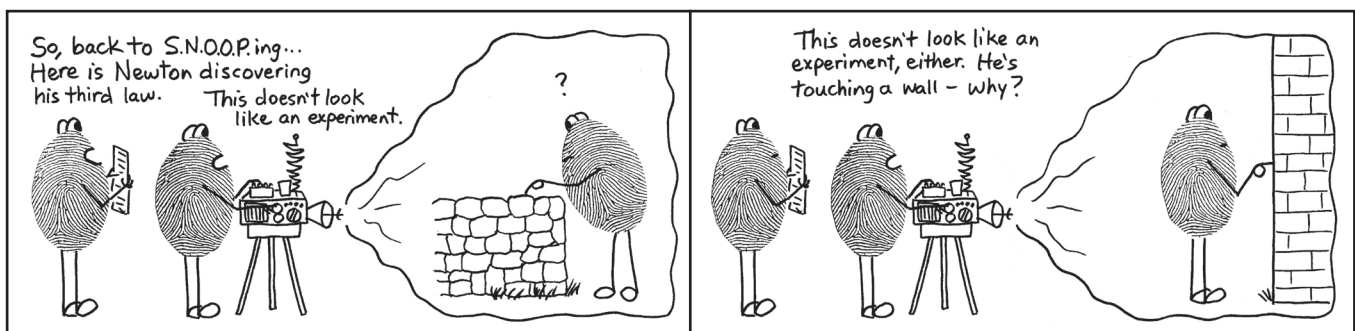
You will need: 2 skateboards (or 2 of anything with wheels that you can stand or sit on), a few friends, heavy backpack

NOTE: If you can't do this experiment yourself, you can watch it as a video lab on the playlist.

Another way to do this lab is to use two identical small object with wheels, and attach identical strong magnets to each, with repelling sides of the magnets facing each other. Bring the magnets very close, then suddenly let go and allow the repelling force to drive the objects away from each other. Then add mass to one of the object but not to the other, and try it again.

In this experiment, force is assumed to be constant. Force will be applied by a physical push.

- 1) Start with one person on each skateboard. Position the boards close together with the riders facing each other. Ideally, the "riders" should have about the same mass. (You also might want to mark the starting point with tape on the floor.)
- 2) Riders should hold up both hands, and touch the other persons' palms. Make sure your arms are in a V-shape, elbows down. You want to be able to extend your arms and give a push.
- 3) Riders will both push against each other at the same time.
- 4) if the riders have the same mass, and they both push equally hard, the skateboards should drift apart the same amount. (The distance traveled will give you an approximation of the acceleration each board experiences. If you marked the starting point on the floor, you can actually measure the distance traveled by each board.)
- 5) Now add one rider to one of the skateboards. You'll have two riders on one board and one rider on the other. Bring the boards together and have the first riders repeat what they did in step 3. Give the same push against each other.
- 6) The result should be that the board with two riders will travel about half the distance as the board with one rider.
- 7) If you can fit three riders on one board, try it with three. If not, try giving a rider a heavy backpack. Repeat step 3. With the added mass, the board should travel an even shorter distance.



After a lot of observation and thinking, Newton came up with his third law of motion. Here is the English translation of what he wrote in Latin in his *Principia* book:

LEX III: *To every action, there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts. Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse will be equally drawn back towards the stone.*

You've probably heard this paraphrase of the third law: **"For every action, there is an equal and opposite reaction."** It is fairly easy to see this law at work when you watch a rocket launch, or when you watch a balloon flying about the room as the air escapes; but the amazing thing is that Newton came up with this idea without watching rockets or balloons. Perhaps he watched someone paddling a boat and observed the paddle going backwards while the boat moved forwards? Or maybe he watched a cannon being fired and saw the cannon jerk backwards as the ball shot out of the barrel? We don't know what got him started thinking about forces, but the more he thought about it, the more he realized that the principles behind forces work everywhere all the time, even if you can see anything moving. The final result was his assertion that as he pressed on a stone, the stone was pressing back. As his feet pressed down on the floor, the floor was also pressing back on his feet. Stones and floors don't move, but according to Newton, they are able to press back. This must be true, said Newton, because basic principles work the same way everywhere, all the time. They aren't true just some of the time, or in particular situations—they are always true all the time. Thus, the basic principles of force must be operating even when we can't actually see them working.

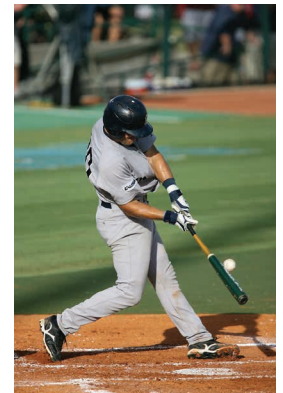


Perhaps Newton observed a cannon jerking backwards as the ball shot out of the barrel.

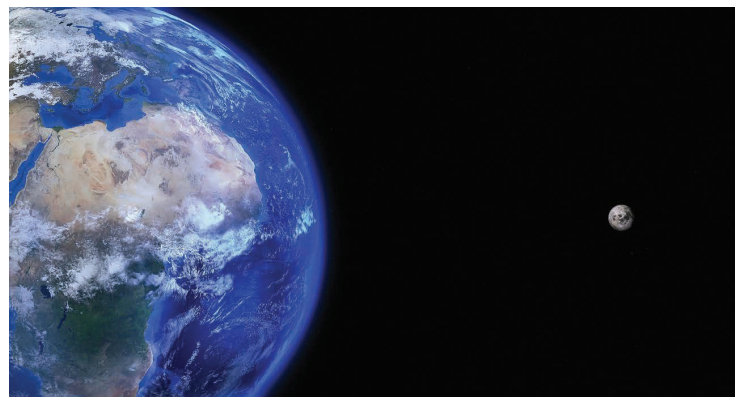
We can elaborate on this third law and identify four parts:

- 1) Forces always come in pairs.**
- 2) These paired forces are always equal in magnitude (strength).**
- 3) The direction of these paired forces is always exactly opposite.**
- 4) The paired forces always occur in two separate bodies (they are not internal forces inside a body).**

The first two ideas are sometimes hard to see. Think of someone hitting a baseball. As the bat smacks the ball and sends it flying, it seems that the bat is providing all the force. The ball goes flying, apparently the receiver of all that force. Where is the opposing force? However, the batter has a different perspective. The batter can feel the force exerted by the ball. His hands and arms feel the "thud" as the bat and ball collide. Rarely, a bat will even crack in half as it collides with the ball, showing the extreme force experienced by the bat. So the ball really does exert an equal and opposite force. But the ball still goes flying off. Obviously, the bat transferred quite a bit of energy to the ball. Where is the opposing force? Newton's third law is about pairs of objects, not single objects. The flying ball is a single object. While it is in the air, we can't apply the third law to it. However, the ball will eventually come into contact with either the fence at the back of the outfield, or the ground, and then the third law can be applied. The fence or the ground will provide a reaction force.

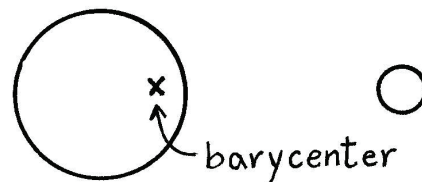


Another strange aspect of Newton's third law concerns gravity. We assume that massive objects have "more gravity" because of their mass. We know that we weigh more on earth than we do on the moon because of earth's greater acceleration due to gravity. However, Newton's third law tells us that forces are always equal and opposite, so if we allow that gravity is a force (setting aside relativity for the moment), this means that the earth and the moon must be pulling on each other with equal and opposite force. This idea doesn't sit well with our intuition. Surely, the earth must be pulling harder



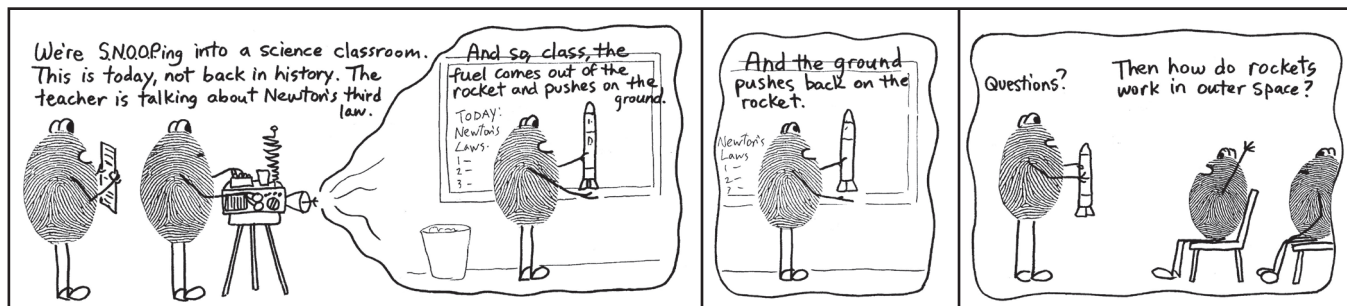
than the moon does because it is more massive. However, we learned previously how to calculate the gravitational force between two objects. We use the formula $F = G(m_1)(m_2)/r^2$. This will give us the gravitational force acting between the objects. They are both experiencing this force regardless of their size. (The formula takes into account the sizes of the objects.) The confusion comes because we don't consider inertia. Though the force might be the same, the effects can be quite different due to the inertia of the objects. The moon has much less inertia than the

earth does because it has less mass. The same force that causes the moon to move around in its orbit is also felt by the earth, but since the earth has much more inertia, it doesn't move very much. However, it does move a little. The common point around which the moon and earth orbit, known as the **barycenter**, is somewhere in the mantle of the earth. So, the moon doesn't orbit the earth—they orbit each other.



We saw this diagram in the first adventure.

And speaking of misconceptions...



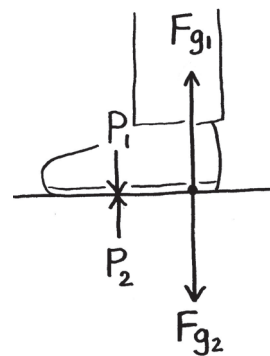
The ground has nothing to do with the opposite reaction that is propelling the rocket upward. If a rocket fires its engines in outer space, the fuel still goes one way and the rocket still goes the other way. The action/reaction pair isn't the rocket and the ground; it's the rocket and the fuel molecules. However, this erroneous statement about the rocket and the ground has actually appeared in teaching materials for kids.

Another common misconception is that the reaction occurs in response to the action. That is, that the reaction occurs a split second later. However, the action and reaction occur at exactly the same time. Likely, the confusion occurs because of the words "action" and "reaction." Perhaps we should call these forces an "interaction" pair.

Even more common than these misconceptions is a belief about something called the **normal force**. We mentioned the normal force in our adventure with friction (page 75). In physics, the word "normal" means "perpendicular." A table leg is normal to the floor (assuming it is not broken). The wooden peg in this photo is normal to the wall. If something is normal to a surface, that means that all the angles between it and the surface are 90 degrees (square corners).



In the context of the third law, the normal force is often described as the opposing force to gravity, meaning the force of the ground pushing up on your feet, neutralizing the force of gravity pulling you down. However, this is technically not correct. There are actually two pairs of opposite forces. The *gravitational* interaction pair is the force of the earth pulling on you (F_{g2}), and the force of you pulling on the earth (F_{g1}). Newton tells us that these forces are equal in magnitude, though this may seem strange since we are so much smaller than the earth. The other pair of forces involve *pressure*, not gravity. There's the pressure that the ground feels as your foot presses down on it (P_1), and the pressure that your foot feels as the ground presses back (P_2). This type of (pressure) force does not need gravity. You can push sideways on a table with your hand and you are applying the same type of pressure to the table that your foot is applying to the floor. (It is this pressure force that bathroom scales actually measure, not the force of gravity. They measure P_2 , how much force is needed to counteract your P_1 force.) However, because these two force pairs are related (the pressure being caused by gravity), P_2 is often renamed F_n , the normal force, and is paired with F_{g2} . We'll learn to label the normal force in some diagrams, but first, some videos and activities.



The P and F forces in the foot diagram represent two very different types of forces. The P forces belong to a category of forces called **contact forces**. Contact forces are caused by (unsurprisingly) physical contact between two objects, and include friction, tension, pressure, and spring force. The other type of force is a **field force**. Gravity is the only field force we will learn about in this book. Electricity and magnetism are also field forces. With field forces, the objects do not have to touch each other. Magnets can attract or repel each other without touching. You can't pair a contact force with a field force. Gravity can't be paired with pressure force. However, engineers find that it works out okay in free body diagrams if you simplify things and just use F_n and F_g (as we did on page 75).

ACTIVITY 7.6: Take a video break!

Let's take a break from reading and watch some videos about Newton's third law. Check the playlist for some short videos that review what we've just learned. (Don't miss the video showing the water-powdered jet pack that lets you "fly" above the water like something in a science fiction movie.)

ACTIVITY 7.7: A third law demo toy: "Clackers"

NOTE: You may want to purchase these instead of trying to make your own.

You will need: a piece of string about 30 cm long (length can vary), two large plastic or wooden beads, marker or pen (optional: metal key ring)

- 1) Push each end of the string through the hole in one of the beads, and then tie a knot in the ends of the string so the beads can hang as shown in photo.
- 2) Measure exactly halfway between the beads and make a dot with a marker.
- 3) Tie a knot exactly where the mark is, or tie the ring at this spot.
- 4) Hold the knot/ring and let the beads hang down. If they are not exactly even with each other, adjust your knot.
- 5) Move your hand up and down until the beads start to clack together. Keep this motion going, increasing the swing of the beads and the strength of their collision.
- 6) With a little practice, you might be able to increase the swing of the balls so much that they go all the way up and clack together over your hands. You can get a very fast over-under-over-under clacking pattern that is double the speed of the regular clacking.

TIP: If you need help figuring out how to use your clackers, search "how to use clackers" on a video platform like YouTube.



ACTIVITY 7.8: Use a skateboard to demonstrate the third law

When you jump, you push on the floor. Newton's third law says that the floor must push back. Does it, really? What if the floor did not push back? What would happen? In this experiment you will push off a surface that doesn't push back.

You will need: a skateboard

- 1) First, stand in a place that gives you plenty of free space for jumping. Do a "standing broad jump" (where you jump from standing position, without any lead-up steps). You can feel yourself pushing against the floor as you begin your jump. But does the floor really push back?
 - 2) Now stand on the skateboard and try the same jump. (Be careful as you jump! Don't hurt yourself.) There will be an equal and opposite reaction as you jump, but in this case you will be able to see that opposite reaction as the skateboard moves backward. How far forward did you jump this time? Likely, you did not move forward at all, or if you did it was a very small amount. The floor had two things that the skateboard did not: friction and inertia.
-

ACTIVITY 7.9: Use your track car to demonstrate the third law

You will need: your track car, at least a dozen pencils, a long piece of cardboard

NOTE: If you don't have a track car, check out the video of this on the playlist.

This activity is very similar to 7.8 with the skateboard.

- 1) First, let your track car move along the floor or table. Newton's third law says that as the car moves forward, the table is receiving an backward force of equal magnitude. However, we can't see this force in the table because the table doesn't move.
- 2) Now place the long piece of cardboard on the table and put all the pencils underneath it, as shown.
- 3) Turn on the track car and set it onto the cardboard. What happens?
The car's wheels will cause the cardboard to move backwards. The car might not move ahead at all.

Let's go back and take a closer look at the batter hitting the baseball. We noted that Newton's second law can be applied as the ball hits the bat. The bat and ball feel equal and opposite forces. However, there is something else going on here, too. The bat was able to reverse the direction of the ball as well as making it sail high up into the air. While the ball was in contact with the bat, it experienced acceleration due to the force of the bat, but after the ball was no longer in contact with the bat, the acceleration stopped. However, the ball still had **momentum**. Momentum is just **movement in a particular direction** and does not require speeding up or slowing down like acceleration does. Was the ball moving at a constant speed while it was flying through the air? This must be the case because it was the bat that was applying the force that caused acceleration. If the ball is no longer in contact with the bat, that accelerating force is no longer being applied. (What the bat did to the ball is called **applied force**.) The ball continues to fly through the air because it has momentum. (Gravity is not slowing the speed of the ball, and friction with air molecules is so small that we can ignore it for all practical purposes.)



There is a very simple formula for determining momentum. The letter "p" is used to represent momentum ("p" is for "pellere," Latin for "push") so the formula looks like this: $p = mv$ (momentum= mass times velocity). Mass is recorded in kilograms, kg, and velocity is in meters per second, so momentum is (kg)(m)/sec. (This formula reminds us of $F=ma$, with the difference being that velocity uses just seconds, whereas acceleration uses seconds per second. So $p=mv$ is sort of like a "step down" from $F=ma$.)

Why bother even mentioning momentum? Partly because it is going to help us explain the behavior of the ball, and partly because Newton didn't actually write down $F=ma$. He actually wrote something more like this: Force equals the change in momentum divided by the change in time (i.e. the amount of time the force is applied). Here is how we write this in mathematical terms:

$$F = \frac{\Delta p}{\Delta t}$$

The triangle is the letter "D" in the Greek alphabet, and is pronounced "delta." The delta stands for "change." This formula becomes more helpful in our discussion of the batter hitting the baseball if we can rearrange it a bit. The rules of algebra say that if we multiply each side of the equation by the same thing, we won't change the equation. So if we multiply each side of this equation by Δt , we get:

$$F(\Delta t) = \Delta p$$

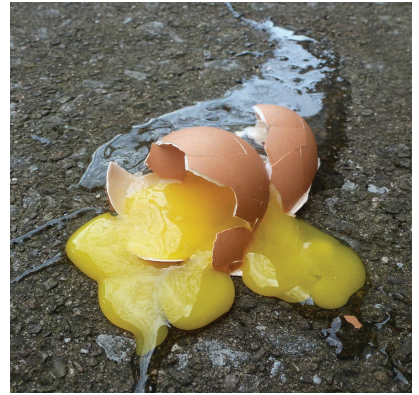
This tells us that the change in momentum (Δp) of the baseball will be determined by the amount of force (F) applied by the bat multiplied by the time interval (Δt) that the bat is in contact with the ball. $F(\Delta t)$ is called the **impulse force**. The size of the impulse will determine the momentum of the ball.

The ball needs to change its momentum from motion towards home plate to motion away from home plate. How big that change will be partly depends on how long (Δt) the bat is in contact with the ball. This is why "follow through" is so important when batting a baseball or hitting a golf ball. The longer the bat or club is in contact with the ball, the more momentum the ball will have.



In certain situations, it is to the team's advantage to have the batter try NOT to give the ball a lot of momentum. They want the ball to stop about halfway to the pitcher's mound. It takes enough time for the catcher to run out and pick up the ball that the runners already on bases will have time to advance to the next base. The batter tries to decrease the momentum on the ball by decreasing the interval of time (Δt) that the bat is in contact with the ball. This is called "bunting" (shown in photo). There is no follow through when bunting. In fact, the batter might even let the bat come backwards a bit, almost like a reverse follow through.

The time interval (Δt) is also important in collisions. The change in momentum in a collision goes from very fast down to zero in an extremely short amount of time. The exact amount of time is very important—fractions of a second can mean the difference between something breaking or not breaking. For example, if you hold up an egg and drop it onto a cement floor, the egg will stop very suddenly due to the stiffness of the floor. Bam! The egg hits zero velocity almost instantly. But if you drop the egg into a bucket of water, the egg will likely survive the fall because the egg's velocity will be slowing down from the time it first touches the water until the time it reaches the bottom of the bucket. Both the floor and the water brought the velocity of the egg to zero, but the floor kept Δt very, very small, while the water allowed for a larger Δt . That little bit of extra time made all the difference.

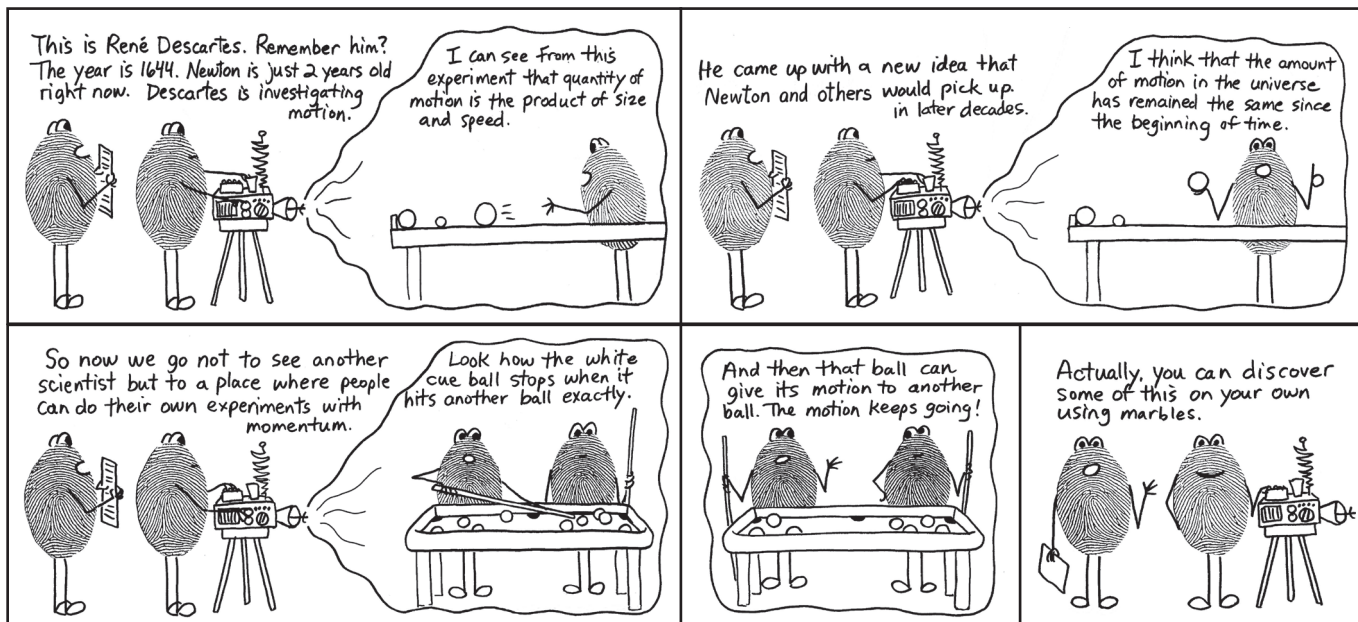


Our visual processing isn't fast enough to be able to appreciate the difference between going to zero velocity in .2 seconds and going to zero in .1 seconds. However, this can be the difference between life and death. Cars are designed to increase Δt as much as possible, slowing the collision time in order to lessen the force of impact. The front of a car (everything from the windshield forward) is designed to crumple. A car that can't crumple is like the egg hitting the floor, going to zero velocity almost instantly. Air bags can add even more Δt . They only add a tiny fraction of a second to Δt , but that split second makes a huge difference in the outcome.

ACTIVITY 7.10: Try a few easy math problems about momentum

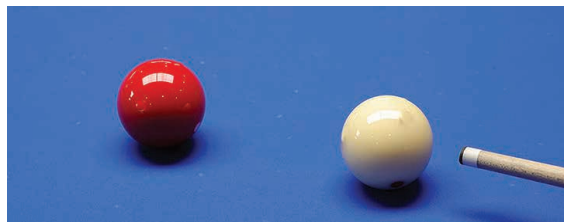
NOTE: Oddly enough, physicists never came up with a name for the units of momentum. For force we have newtons, but for momentum we have to write out "kg(m)/sec." There are so many unit names in physics—newtons, joules, watts, volts, amps—you'd think they would have come up a name for momentum units. But nope, they didn't.

- 1) A golf ball has a mass of .05 kg. If a golfer hits the ball giving it a velocity of 30 meters per second, what is the momentum of the ball? _____ (Use $p=mv$. Your answer will be in kg(m)/sec.)
- 2) A baseball has a mass of 145 grams. A pitcher throws the baseball hard enough to give it a momentum of 3.75 kg(m)/sec. What was the velocity of the ball? _____ (Use $p=mv$. Don't forget to convert grams to kg.)
- 3) A tennis player hits a ball and gives it a momentum of 1.5 kg(m)/sec. (The change in momentum is from zero to 1.5, so we can just use the number 1.5.) The racket is in contact with the ball for .2 seconds. (The change in time is from zero to .2 seconds, so we can just use the number .2.) How many newtons of force will be the result? _____ (Use $F= \Delta p/\Delta t$)
- 4) A baseball pitching machine can pitch balls with a force of 50 N. We know that the momentum of the balls as they come out of the machine is 5.5 kg(m)/sec. How long (Δt) were the balls in contact with the pitching mechanism? _____ (Use $F(\Delta t)= \Delta p$, or you can rearrange it by dividing both sides by F, to make $(\Delta t)= \Delta p /F$. Your answer will be in seconds.)
- 5) An average hockey puck weighs about 160 grams. If a puck is coming at the goalie with a momentum of 3.2 kg(m).sec, what was the velocity of the puck as it left the stick of the player who hit it? _____ (Don't forget to convert grams to kg. Use $p=mv$. Your answer will be in m/sec.)
- 6) A car's momentum went from 5 kg(m)/sec down to zero in one quarter of a second because it hit a wall. How many newtons of force was this collision? _____ (Use $F= \Delta p/\Delta t$.)



One idea we'll be exploring in the next activity is called **conservation of momentum**. (The word "conservation" means to preserve or to prevent change.) This is the idea that occurred to Descartes, though he could not prove it. The idea was debated and refined through the centuries and then became an established law of physics somewhere in the middle of the 1800s. Conservation of momentum is a key concept when studying collisions. For a collision occurring between two objects in an isolated system (meaning they are not being affected by any other forces), the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision. The momentum lost by object 1 is equal to the momentum gained by object 2. This is what we will be looking for in the following activity, though we might be using more than two objects. Our "system" will be two or more marbles colliding with each other inside a paper track.

But, wait—before you start the next activity, here is something else to look for: **elastic and inelastic collisions**. In elastic collisions, not only is momentum conserved, but motion energy (**kinetic energy**) is also conserved. Billiard balls are designed to give elastic collisions. (Technically, not perfectly elastic, though. Almost no real-world collisions are perfectly elastic. There is always a small amount of energy lost due to friction, sound and heat being produced in the collision.) If the cue ball hits another ball exactly at its center, almost all of the momentum and kinetic energy will be transferred to the other ball. "Motion in" equals "motion out," even if the motion gets divided up between two or more balls. When you "break" at the beginning of the game, the momentum and kinetic energy of the cue balls gets distributed randomly to all of the balls because they are all in contact with each other. (Again, these are not perfectly elastic collisions, but we go ahead and give them the benefit of the doubt and call them elastic anyways.)



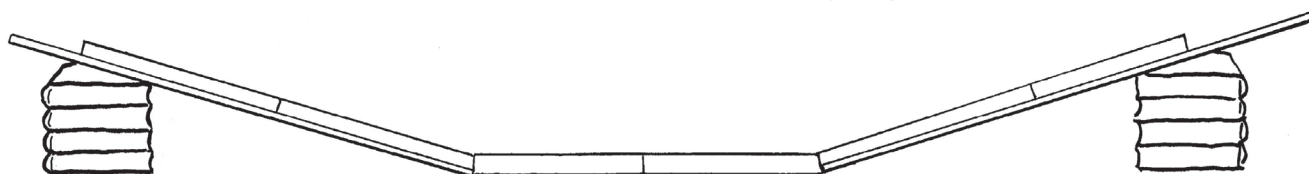
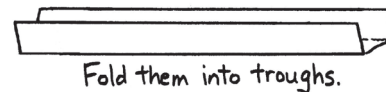
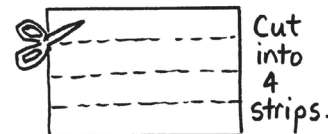
Inelastic collisions must conserve momentum (it's the law!) but they can have an observable loss of kinetic energy. Imagine throwing a ball of clay a wall. The kinetic energy disappears because the wall didn't move and the clay ball didn't bounce back. How is momentum conserved? If you look at the clay ball after it hits the wall, you'll see that its shape is greatly deformed. The momentum went into the molecules of the clay. Inelastic collisions result in the colliding objects being joined together. The classic example of this is two train cars going in the same direction, with the rear car going much faster. The rear car catches up, then attaches to the forward car and the two keep going along together. You might see something like this happening with your marbles.



ACTIVITY 7.11: Observing collisions

You will need: about 10 marbles, 2 sheets of card stock, scissors, tape, meter (or yard) sticks, two stacks of books
Optional: other small balls you might want to try

- 1) Cut a sheet of card stock in half the long way. Then cut these halves in half the long way. Now you will have 4 long strips. Do this to the other piece of card stock, also.
- 2) Fold the strips in thirds, the long way, so that you have paper troughs perfectly sized for holding marbles.
- 3) Tape 6 of the troughs end to end, making sure the transitions between troughs are smooth (no bumps to stop marbles from rolling smoothly).
- 4) Tape the two end troughs to the meter sticks, with two troughs in the middle, as shown in the diagram below. Place a stack of books under each meter stick, making sure the stacks are the same height.



- 5) Now you are ready to begin your experiments. You will be trying out as many combinations as you can, with different numbers of marbles rolling down the troughs or sitting in the middle trough. You will need to make sure that when you let marbles go on the ramps that they are released at exactly the same time. For many of your experiments, you'll also want to make sure the marbles are released from the same point on the ramp, but you can control acceleration by the height of the starting point.

Here are some suggestions for combinations to try:

left ramp	middle	right ramp
1	0	1
2	0	2
1	2	0
1	3	0
1	4	0
1	5	0
1	2	1
2	2	0
2	3	0
2	4	0
3	4	0

- 6) Can you engineer an inelastic collision?

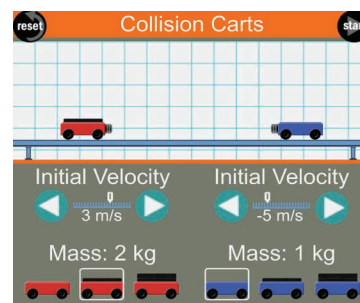
In an inelastic collision, both objects are in motion going the same way, but the rear object is going faster. After the collision they stay in contact with each other and keep going as if they were a single object.

None of your collisions will be perfectly elastic or inelastic, of course. We can't get away from friction and all the other little imperfections of real life.

ACTIVITY 7.12: "Collision Carts" an online simulation

You can experiment with momentum and collisions by going to:
<https://www.physicsclassroom.com/Physics-Interactives/Momentum-and-Collisions/Collision-Carts/Collision-Carts-Interactive>

Notice that the velocities are shown as (+) and (-) because velocity is a vector, meaning it has direction as well as magnitude. They decided to indicate direction using (+) for direction to the right, and (-) for direction to the left.



ACTIVITY 7.13: Review questions

(Answers are in the teacher's guide section.)

- 1) TRUE or FALSE? Newton wrote $F=ma$ in his Principia.
 - 2) TRUE or FALSE? Newton wrote in English.
 - 3) What measurement unit is kg(m)sec^2 ? _____
 - 4) What is measured using kg(m)sec ? _____
 - 5) What is the average mass of a pineapple? _____ And how many newtons is this? _____
 - 6) Does $F=ma$ work in zero gravity (free fall)? _____
 - 7) Does increasing acceleration increase force? _____
 - 8) How much does a 100 kg person weigh in newtons? _____
 - 9) TRUE or FALSE? The earth and the moon pull on each other with equal strength.
 - 10) TRUE or FALSE? The earth has more inertia than the moon.
 - 11) The center of gravity around which two bodies orbit is called the _____.
 - 12) TRUE or FALSE? The action and reaction occur at exactly the same moment.
 - 13) What word do physicists use for perpendicular? _____
 - 14) A soldier is standing at attention. How many action/reaction pairs are there in his feet? _____
 - 15) There are two kinds of forces. When objects touch, these are _____ forces. When objects do not touch, these are _____ forces.
 - 16) $F(\Delta t)$ is the: a) interval of time b) impulse force c) momentum d) velocity e) applied force
 - 17) To prevent something from breaking, do you want Δt to be large or small? _____
 - 18) Who was the first person (as far as we know) to suggest that momentum is conserved? _____
 - 19) TRUE or FALSE? Conservation of momentum occurs even if the objects in a collision stop moving.
 - 19) Can you ever witness a 100% completely elastic collision? _____
-

ACTIVITY 7.14: Matching definitions

(Answers are in the teacher's guide section.)

- | | |
|----------------------------|--|
| 1) impulse force _____ | A) the measure of how resistant something is to a change in velocity |
| 2) momentum _____ | B) the perpendicular force of an object pressing on a surface |
| 3) conservation _____ | C) the highest velocity an object can achieve while falling |
| 4) field force _____ | D) the measure of how fast something is going |
| 5) mass _____ | E) the measure of how fast and in which direction something is going |
| 6) speed _____ | F) moving |
| 7) velocity _____ | G) how large or small something is |
| 8) terminal velocity _____ | H) no kinetic energy is lost |
| 9) acceleration _____ | I) force times the time interval in which the force is applied |
| 10) normal force _____ | J) velocity multiplied by mass |
| 11) inertia _____ | K) the increase in velocity |
| 12) kinetic _____ | L) the tendency of something to remain as it is |
| 13) weight _____ | M) measuring mass while in a particular gravitational field |
| 14) magnitude _____ | N) no change, or preventing change |
| 15) elastic _____ | O) force that operates without objects touching each other |

CHAPTER 7:

Answers to 7.3:

- 1) 607.6 N (Multiply 62 kg by 9.8 m/sec²)
- 2) 600 N (Multiply 20 kg by 30 m/sec².)
- 3) 50 kg (Divide 490 by 9.8 to get 49.999=50)
- 4) 9 m/sec² (Divide 490 by 15.)
- 5) .0784 N (Multiply .008 kg by 9.8 m/sec².)
- 6) 1 N (Multiply .2 kg times 5 m/sec². 10 m in 2 sec. is 5 m in 1 sec.)
- 7) 5,100 N (Multiply 1,700 kg by 3 m/sec².)
- 8) 588 N (Multiply 60 kg by 9.8 m/sec².)

Answers to 7.4:

Note: If your students don't know how to do unit conversions, this is a good opportunity to introduce the topic.

$$\frac{.5 \cancel{\text{kg}}}{1} \times \frac{.45 \text{ kg}}{1 \cancel{\text{kg}}} = .225 \text{ kg}$$

- 1) 2.2 m/sec². (F=1, m= .45 kg. Divide 1 by .45 to get 2.2)
- 2) 17.7 m./sec² (3 oz.=.25 pound. Multiply .45 kg by .25 to get .1125 kg. So you have 2=(.1125)(a). Divide 2 by .1125.)
- 3) 5,000,000 N (a=200/10 = 20 m/sec². Then do mass times acceleration: (250,000 kg)(20 m/sec²) = 5 million N.)
- 4) 12 m/sec² (8 ounces = .5 lb, then multiply .5 times .45 to get .225 kg. Then (2.7)=(.225)(a). Divide 2.7 by .225 to get 12.)
- 5) 2 kg (Convert .25 km to 250 m. Then find (a) by dividing 250 by 50 seconds. So (a)=5 m/sec². Then 10=5a, so a=2.)

Answers to 7.10:

- 1) 1.5 kg(m)/sec (Multiply .05 kg by 30 m/sec.)
- 2) 20 m/sec (Convert grams to kg. 145 g = .145 kg. Then (2.9)=(.145)(v). Divide 2.9 by .145 to get 20 m/sec.)
- 3) 7.5 N (1.5 divided by .2.)
- 4) .11 seconds (50(Δt)=5.5 Divide both sides by 50. 5.5 divided by 50 is .11)
- 5) 20 m/sec (3.2=(.16kg)(v) 3.2 divided by .16 is 20.)
- 6) 20 N (5 divided by .25.)

Answers to 7.13:

- 1) False, he said things that essentially meant the same thing as this formula, but he did not actually write "F=ma."
- 2) False, he wrote in Latin
- 3) newtons, N
- 4) momentum
- 5) 1 kg, 9.8 N
- 6) yes
- 7) yes
- 8) 980 N
- 9) True
- 10) True
- 11) barycenter
- 2) True
- 13) normal
- 14) 2 pairs
- 15) contact, field
- 16) b
- 17) large
- 18) Descartes
- 19) True
- 20) no

Answers to 7.14:

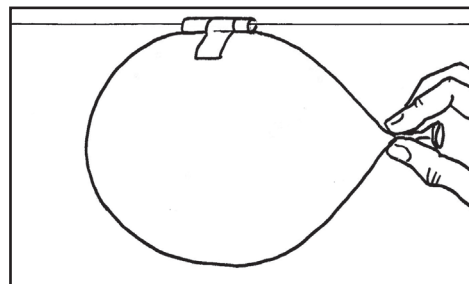
- 1) I 2) J 3) N 4) O 5) A 6) D 7) E 8) C 9) K 10) B 11) L 12) F 13) M 14) G 15) H

ACTIVITY 7.15: Ye Old "Balloon Rocket" Activity

This activity is so well-known that I decided not to include it in the text. I figured that probably most of the students using this book have already done, or at least seen, this activity. It is a classic activity for elementary grades. It's pretty basic. Blow up balloon, let it go, it travels along string.

You will need: a balloon, a straw, scissors, tape, fishing line

- 1) Cut a small piece of straw, about 3 to 4 cm long.
- 2) Thread the straw onto the fishing line.
- 3) Stretch the fishing line across a room, and tape the ends securely so that the line stays taught.
- 4) Cut some pieces of tape and have them ready.
- 5) Blow up the balloon. Do not tie it shut!
- 6) Hold the balloon right under the piece of straw and tape the straw to the balloon.
- 7) Pull the balloon to one side of the string, and then let go.
- 8) The balloon will "rocket" across the room, along the fishing line.
- 9) Make sure the students understand that the opposing forces here are the air molecules escaping out the back of the balloon, and the kinetic motion of the balloon traveling along the line.



ACTIVITY 7.16: Another online collision simulation program

Go to: <https://phet.colorado.edu/en/simulations/collision-lab> (This is a more complex simulation than the Collision Carts.)

ACTIVITY 7.17: Experiment with a "Newton's Cradle" toy

You will need: a Newton's cradle, easily purchased from any online science store or from Amazon

Newton's cradle was not invented by Newton himself, only named in his honor because it demonstrates the third law. The metal balls provide very elastic collisions. This toy is a lot of fun, but be aware that you must be very gentle with it. I've found that younger students left unattended will find a way to tangle the strings somehow. The strings are made of fishing line (or something very like it) so they are plenty strong for ordinary use of the toy, but will break if the toy is misused.



ACTIVITY 7.18: Field trip to elevator to stand on scale and see weight change

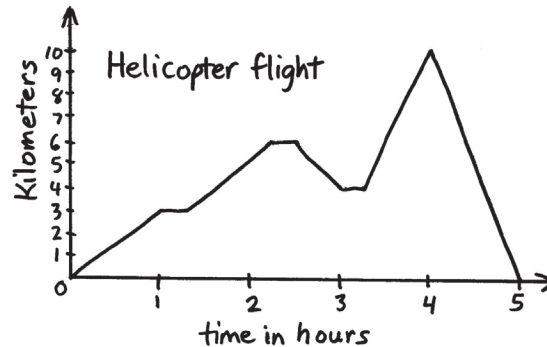
You will need: a bathroom scale, an elevator (one that goes fairly high and fairly fast)

- 1) Enter the elevator, place the scale on the floor and stand on the scale. Notice the weight reading.
- 2) Press the button to go to the highest floor. (Hopefully, you can get a straight shot up and the elevator won't have to pick up passengers.)
- 3) Watch the scale very carefully. What happens to the weight reading as you go up?
- 4) When you get to the top, press the button to go down again.
- 5) Watch the scale very carefully again. What happens to your weight reading?

On the way up, you will experience additional acceleration (in addition to gravity), so your weight reading should go up. On the way down, you will experience negative acceleration (which will counter gravity), so your weight reading will go down.

CHAPTER 8:

Answers to 8.2:



Answer to 8.3:

