

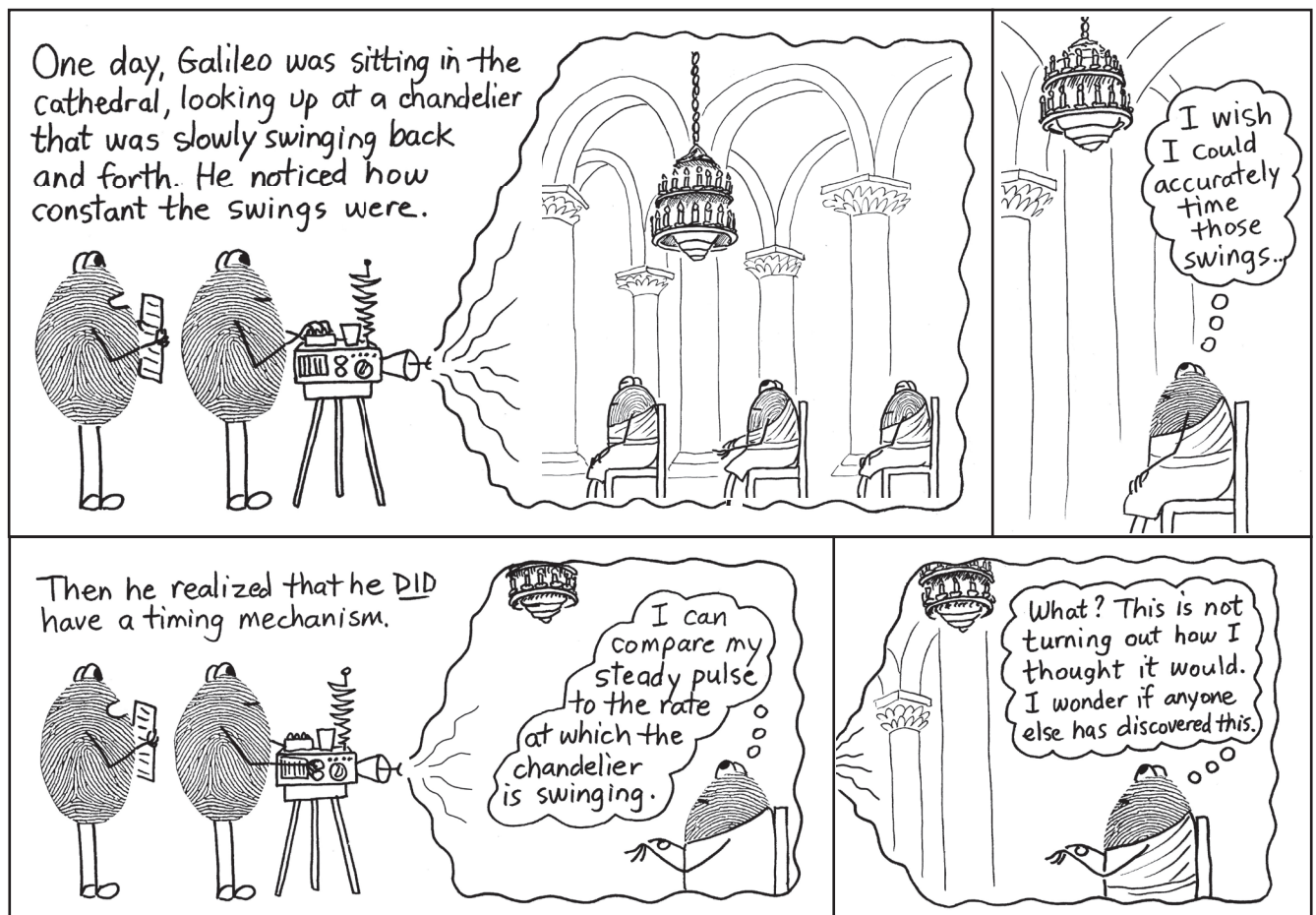
## Adventure 3: Swinging with Galileo

Our time travel adventure is going to fast forward now, from about 250 BC to about 1600 AD. We are going to send our volunteers to spy on one of the most famous scientists of all time: Galileo Galilei. He was born in the city of Pisa, in what we now call the country of Italy; but back in his time, each Italian city was like a tiny, independent country. Sometimes these city-states even fought wars against each other.

The city of Pisa (which you will remember from our center of gravity adventure) belonged to the larger city of Florence. Florence was, and still is, a very cultural city, filled with artists and musicians. Since Galileo's father was a musician, he decided to move his family to Florence when Galileo was 8 years old. Thus, Galileo grew up surrounded by beautiful Renaissance buildings and sculptures, reading Latin and Greek literature, and listening to the finest music available at that time. His father was very interested in music theory—the mathematical structure of music. He taught Galileo much about both the mathematical basis of music. Galileo was successful at everything he studied: Greek, Latin, literature, poetry, music, and art. Though none of his paintings have survived till this day, we know from the writings of people who knew him that he was a very talented artist. It was also said that he could play the lute (a guitar-like instrument) as well as the professionals of his day.

Galileo's father knew all too well how hard it was to support a family on a musician's or artist's income, so he told Galileo that he had to study medicine and become a doctor. Galileo would have preferred to become a priest or a monk at that point in this life, or maybe an artist, but he had to conform to the wishes of his father and he was packed off to study medicine at a school in Pisa.

While studying in Pisa, Galileo often went to church services in the cathedral.



What surprised Galileo? You can find out for yourself by doing the experiment on the next page.

### ACTIVITY 3.1: Make a swinging chandelier

**You will need:** a copy of the chandelier page (opposite) printed onto card stock, scissors, a pin, a stop watch or clock

- 1) Print the following pattern page onto card stock. Trim off the chandelier piece, but make sure to keep the other two stripes, as you will be using them later. If you are working from a paperback copy of this book (instead of a digital book), you can cut out the chandelier page and glue the chandelier strip to thin cardboard or thick paper. (You can also download patterns here: [www.ellenjmchenry.com/discoveringmotion](http://www.ellenjmchenry.com/discoveringmotion))
- 2) Push a pin through the paper right at the top of the chandelier chain, about a centimeter from the top. Wiggle the pin a bit so the hole enlarges just slightly. The strip of paper should be able to swing freely from the pin. Make sure the paper is hanging straight and is not curled at all. Also, make sure you are in a place where the air is very still. If you are near a heater, the paper can be blown about by warm currents of air. You can either hold the pin with your fingers (if you can hold it very still), or you can tape one end of the pin to the edge of a table. Or, if you have a bulletin board handy, you can stick the pin into it.
- 3) First, we will do an experiment without a timer. You will pull the chandelier to the side and let it go and then watch it until it stops. (Don't pull it too far to the side. If you imagine that the chandelier is the hour hand of a clock, pointing at 6 o'clock, pull it to no more than 4 or 8 o'clock, maybe a little less.) As you watch it swing, the important thing to notice is the "beat" of the back-and-forth motion. Keep time with it in your head—perhaps sing a song in your mind, with the chandelier providing the "beat." Notice if the song slows down or stays steady.
- 4) Now we will use the timer. Get the stop watch or clock ready. You will count how many times the chandelier swings in 10 seconds. Pull the chandelier to one side and then let it go and start it swinging. (Again, don't pull it too far to the side, no more than the 4 or 8 o'clock position.
- 5) Now try it again, but this time pull it to only the 5 or 7 o'clock position. Time for another 10 seconds.
- 6) Try it again at least one more time.
- 7) What did you discover about the time of the swings? Did the little swings seem to take as long as the big ones? If you were accurate with your timing, you should have found the results to be very close to identical, if not identical. This is what Galileo discovered. Those tiny swings (as the chandelier is almost stopping) take just as much time as the bigger ones!



It appears that the distance the pendulum travels back and forth does not affect how long it takes to complete a full swing. You came to this conclusion and so did Galileo. This is entirely true if you don't start the pendulum swinging too high. As long as the swings are relatively small (less than about 10 degrees from the vertical starting point) the timing is so accurate that a pendulum can be used as a time-keeping device inside a clock. However, for larger swings, pendulums are not perfect time-keepers. It turns out that if you release a pendulum from a very high point, those first couple of swings take more time than the smaller ones. Mathematicians can calculate exactly how much difference there is between large swings and small swings. This is what their equation looks like:

$$T = 2\pi\sqrt{\frac{L}{g}} \left( 1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \frac{173}{737,280}\theta_0^6 + \dots \right)$$

Hmm... let's not delve into that one! Let's just say there is some difference between big swings and little swings, and big swings are not useful for time keeping. In the next activity, we will see this difference.

This kind of equation is called an "infinite series."

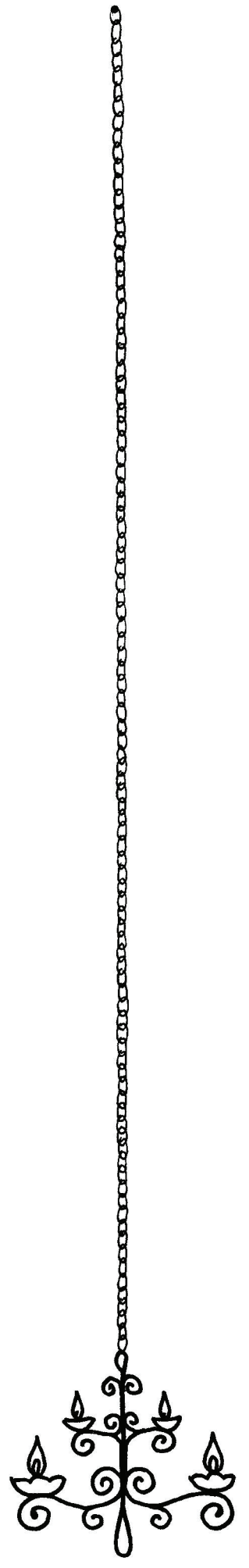


### ACTIVITY 3.2: Wide swings compared to narrow swings

**You will need:** a long object such as a meter stick (yard stick) or two rulers taped together, or any long skinny object you have at hand such as a golf club or ball bat, a piece of string, scissors, tape (duct tape works well), and your timer

- 1) Tape a loop of string to the end of your long object. Hold it so that it can swing freely. Make sure to not to move while holding the object; you don't want any body motion to influence your results.

(continued on next page)





- 2) Pull the object up to a very high starting point, almost straight up. Get your timer ready. Let go and time how many swings it does in 20 seconds.
- 3) Now try it again, but only pull back the object a tiny bit so the swings will be small. Time another 20 seconds.
- 4) Did the very large swings take a bit longer than the small ones?

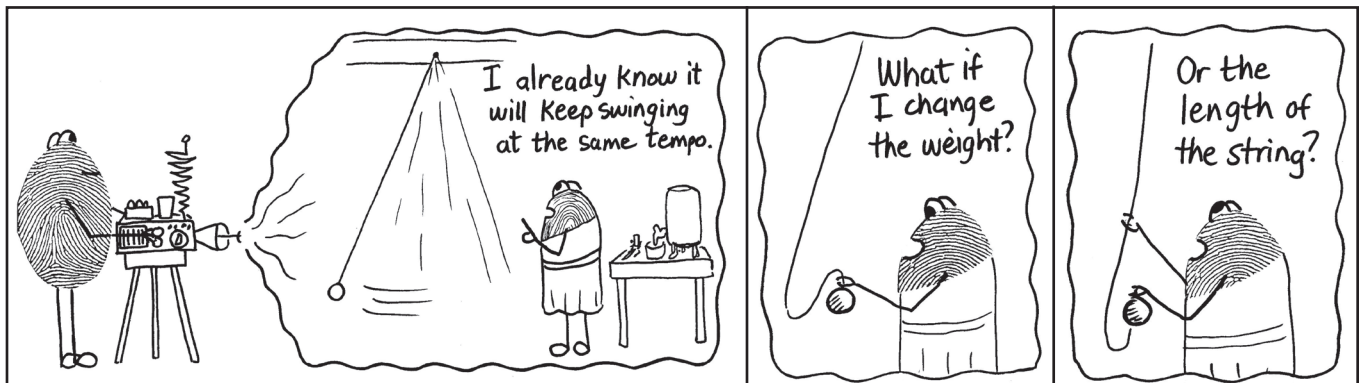
You'd think that after making this interesting discovery, Galileo would have realized that a pendulum might be used as the timing mechanism for a clock. But he didn't. Instead, perhaps because he was in medical school at the time, he thought about reversing the situation and instead of using a pulse rate to time the pendulum, perhaps a pendulum could be used to time pulse rate. He designed a simple pendulum device for doctors that he called a "pulsilogium." With this device, doctors could determine an exact measurement of how fast a patient's heart was beating. Galileo took great delight in designing and building practical tools. Years later, he would finally realize the potential of the pendulum for time-keeping and would sketch a design for a pendulum clock. Sadly, this was right before he died and he never got to build the clock. But this is getting ahead of our story.

Galileo was very busy trying to keep up with his university classes. He wanted to take a math class in addition to his medical classes, but his father had made it clear that he should not waste time studying math while he was trying to finish his medical degree. But then, one day, when Galileo was at home during a school break, a visitor came to the house. Ostilio Ricci (*ree-chee*) was a friend of Galileo's father, and just happened to be a professional mathematician. Ricci struck up a conversation with Galileo and the topic eventually turned to math. Ricci could see that Galileo was extremely interested in math, and told his father that he should let Galileo come to a geometry class he was teaching. Galileo's father said "absolutely not!" but it was too late—Galileo now knew about the math class. He began sneaking into the back of the classroom and hiding. Ricci was teaching from Euclid's *Elements*. (Yes, Euclid's geometry textbooks were still being used even though they were 1,500 years old!) Galileo eventually found a copy of the *Elements* in a book shop in Pisa, and began studying it on his own. He took the book home during school breaks but kept it hidden under a pile of medical books. When his father wasn't looking, he'd pull out the geometry book, perhaps even hiding it inside the cover of a medical book.

Because Ricci was a friend of his father, Galileo had opportunities to talk with Ricci from time to time, outside of the classroom, of course. Galileo's questions about geometry so impressed Ricci that he asked Galileo who his math teacher was. Galileo was silent for a while, then confessed that he had been sneaking into Ricci's classes. Ricci laughed and told Galileo that he was welcome to attend any of his lectures, and to come right in and sit anywhere he wanted to. He also began speaking to Galileo's father, telling of the boy's incredible aptitude for math. Ricci could tell that Galileo was never going to be a doctor—he was a born mathematician. At first, Galileo's father was furious. But after months of coaxing, he finally gave in and allowed Galileo to switch his major from medicine to math. Galileo promised his father that he would find a way to make a living doing math.

Ricci then introduced Galileo to the writings of Archimedes. Now Galileo had a role model that suited him very well. Like Archimedes, Galileo would be not only a mathematician but an inventor, as well.

Our time machine is now going to skip to a little bit later in Galileo's life, when he has already been a professor of mathematics for several years, and has become somewhat famous in northern Italy. At this point in his life, he was able to spend some time doing experiments. He remembered how interested he'd been in pendulums and decided to pick up these thoughts where he'd left off years ago.

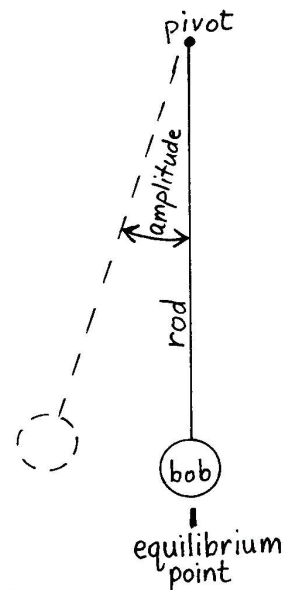




Once again, you can discover these things at the same time that Galileo is discovering them (in our story), but first, here are some helpful vocabulary words.

The weight on the end of a pendulum is called the **bob**. The place where it hangs is usually referred to as the **pivot** and the string is called the **rod**. When the pendulum is hanging straight down and not moving at all, it is said to be at its **equilibrium point**.

Once the pendulum starts to move, there are names for the aspects of its movement. The size of a swing is called the **amplitude**. The amplitude is measured in degrees—the same degrees that you use to measure angles in geometry. One complete swing back and forth is called a **cycle**. The time it takes for a pendulum to complete one cycle (a complete swing) is called its **period** (meaning the period of time it takes). The number of cycles (swings) per second (or per minute) is called the **frequency**.



### ACTIVITY 3.3: What happens when you change the weight of the bob?

**You will need:** a piece of thread about 40 cm (16 in.) long, 10 paper clips, your timer

#### PART 1:

- 1) Tie a paper clip onto the end of the thread. Open the clip a bit to make a "hook." Put another paper clip onto the hook.
- 2) Use the timer to record how many swings your pendulum makes in 20 seconds.
- 3) Add a few more paper clips to the hook and then time it again.
- 4) Add a few more paper clips and time it again.
- 5) Load the last of your clips onto the hook and time it again.
- 6) What can you conclude about how weight affects the frequency of a pendulum?

**NOTE:** You may have some slight differences in the numbers. In all science experiments, there is something called the "margin of error." Since you are human, you are not perfect. Even though it seemed to you that you released the pendulum at the exact moment you heard the word "go," the truth is that you released it a fraction of a second differently each time. You were also off by a fraction of a second each time you called out, "Stop." There may also have been small air currents that were not noticeable to you but nevertheless affected your outcomes each time. Life's little irregularities made their mark on each of your experiments. Therefore, we must expect small errors. Counts that were only one number off (maybe even two?) can be considered identical results.

#### PART 2:

We will repeat this experiment, but instead of counting how many cycles it completes in 20 seconds, you will count how many cycles it completes from the time it starts swinging until it stops.

- 1) Load one clip onto the hook. Pull it back just a bit (low amplitude) and let go. Count how many cycles it makes until it comes to a complete stop.
  - 2) Add all the rest of the clips and repeat. Make sure you pull back the pendulum the same amount. Count how many cycles it makes until it comes to a complete stop. (To speed up this experiment you can do both 1) and 2) at the same time using different pendulums.)
- How does the weight of the bob affect the motion of a pendulum? Does it swing longer?

This makes me feel like I'm fishing. But with paper clip worms!



#### BONUS EXPERIMENT: The importance of center of mass

- 1) Take 10 paper clips and attach them end to end, so that they hang down in a long chain.
- 2) Loop a piece of thread through the top paper clip and tape the thread to the edge of a table so that the top clip in the chain is right below the table, almost touching it. Make sure the chain can swing freely.
- 3) Hang your pendulum from PART 1, putting 9 clips on the hook. Each of our pendulums now has total 10 clips. Make sure the thread is taped so that this pendulum is exactly the same length as the chain of clips.
- 4) Pull back both pendulums the same amount, and notice any differences in their swing. Is their period the same?
- 5) Watch them until they stop. Does one swing longer? Can you adjust the length of the thread chain so its period matches the chain? This experiment demonstrates the importance of center of mass. It makes a difference if the mass is distributed evenly throughout the rod or occurs mostly at the bottom of the rod.

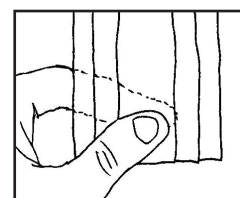
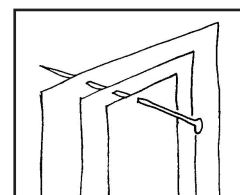
So the weight of the bob has no effect on the period of a pendulum; it only affects how long it will keep swinging. Many people find this surprising. They assume that adding more weight will slow it down. They also think that gravity pulling on the bob is what eventually brings the pendulum to a stop. Not so. Gravity does not slow down a pendulum. What slows down a pendulum is what slows down any moving object: friction. Where can friction be found in a pendulum? There aren't many possibilities, are there? In our chandelier pendulum, the pin was rubbing on the inside of the tiny hole in the paper. Doesn't seem like that would be enough friction to stop the pendulum, but it eventually does. There is another source of friction that is not obvious at all: the friction of air molecules rubbing against the pendulum as it swings. Air molecules don't seem like they would cause much friction, but they do contribute enough friction to be a factor we can't overlook. A friction-free pendulum would go on swinging forever!

### ACTIVITY 3.4: Find out what speeds up or slows down a pendulum

**You will need:** scissors, your paper chandelier, and the rest of the page it was cut from.

#### PART 1: Does the width of a pendulum affect how fast it swings?

- 1) Cut along the remaining line on the sheet from which you cut the chandelier strip. Now you should have three strips of paper that are the same length but different widths. (The thinnest strip will be the chandelier.)
- 2) Place the medium and large strips under the chandelier strip. Line them up so that the tops and bottoms match up perfectly, and so that they are also centered perfectly. Put the pin through the hole at the top of the chandelier and poke straight through all three strips. Wiggle the pin so that the hole is large enough that all three strips swing freely.  
Let the strips hang straight down. Spread the tops apart just slightly on the pin so that when they swing they will not be touching each other.
- 3) Pinch them in the middle (so that they won't slip apart) and pull them back. Then let go. They should all start swinging at precisely the same moment.
- 4) What happens? Do they basically swing at the same rate? (After they swing for a while, there will probably be more difference in their swings. This may be due to the greater effect of air resistance on the wider pendulums, or to some other cause.)



#### PART 2: Let's try varying the length of the pendulums

- 1) Use those wide strips to cut three more narrow strips the exact width of the chandelier strip.
- 2) Leave the chandelier strip full length. Cut the other three strips so that you have one that is  $\frac{3}{4}$  the length of the chandelier, one that is  $\frac{1}{4}$  the length of the chandelier, and one that is  $\frac{1}{4}$  of the length.
- 3) Line these strips up so that their tops match exactly. Push the pin through that hole in the top of the chandelier chain one last time, then poke it through the other three strips. You should now have four strips of different lengths hanging from your pin.
- 4) Spread the strips apart so that they can swing freely without touching each other. (If the strips are curled a bit lengthwise, run your fingers down them a few times and they should straighten out.)
- 5) Use your finger to pull back all four at once, then let go. What happens? Does the length of a pendulum affect how fast it swings back and forth?



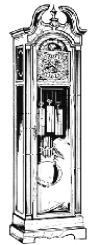
Now you know how to make pendulums go faster or slower: adjust the length. You know how to make a pendulum swing longer: add weight to the bob. You are now ready to make a reliable time-keeping pendulum. Can you make a pendulum that ticks off seconds, like a clock?

*Galileo eventually invented a pendulum clock, but, sadly, it was after he went blind. His son drew sketches of his clock idea, but both he and Galileo died before the clock could be built. A Dutch scientist name Christian Huygens picked up the idea and was the first to build a pendulum clock.*

### ACTIVITY 3.5: What happens when you change the weight of the bob?

**You will need:** your thread and paper clip hook from activity 3.3, another paper clip, and a way to count off seconds (you might try [www.metronomeonline.com](http://www.metronomeonline.com))

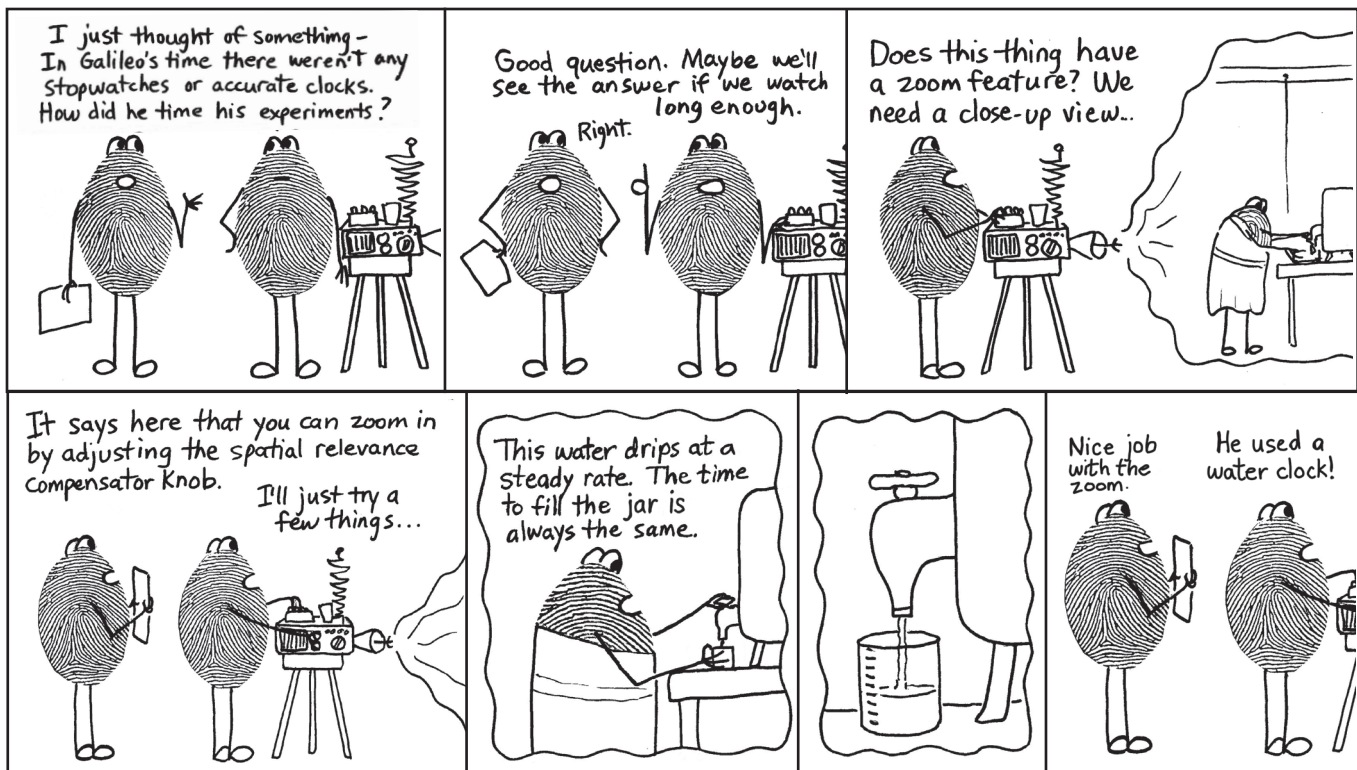
- 1) Measure the thread. If it is shorter than about 35 cm, attach a new piece of thread that is more than 35 cm.
- 2) Wind the top of the thread around the other paper clip, as shown in diagram on right. This will give you a way to quickly and easily adjust the length of the thread.
- 3) Hold the clip and the wound thread so that it won't slip while the pendulum is swinging. Start the pendulum swinging and compare a complete cycle (back and forth) to the seconds ticking on the metronome or stopwatch.
- 4) Adjust the height of the bob, up or down, to better match the ticking seconds. Keep adjusting until you get your pendulum to tick of accurate seconds. The longer you let the pendulum run, the more information you will get about its accuracy. It might seem very accurate for the first few seconds, but after several dozen seconds it might be out of sync.



**Here is an interesting question:** Is it possible to make a working pendulum clock for a doll house? (How long would the pendulum have to be if you wanted the clock to tick off seconds?)

**BONUS ACTIVITY:** Turn on some music and try to make your adjustable pendulum keep time to the music.

As Galileo worked with pendulums, he began to sense that their behavior was based on mathematical principles. Perhaps there was a mathematical relationship between the period of a pendulum and the weight of the bob or the length of the rod? He didn't know what he was looking for—he just started experimenting and writing down measurements.



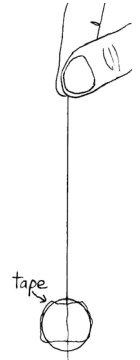
It is thought that Galileo also used the stars as time keepers. It is hard to imagine someone keeping a pendulum going for 24 hours, until a star reappeared in the same place the following night, but some science historians claim that this was, indeed, done. You can perform Galileo's experiment (minus the water clock and the 24-hour vigil), and discover some pendulum math for yourself. However, we've streamlined the process so that you don't have to start from scratch like Galileo did; your experiments will lead straight to a conclusion. Galileo's experiments probably took several months to complete; yours will only take a few minutes.



### ACTIVITY 3.6: Discover some pendulum math

**You will need:** a long piece of thread (90 cm or so), a coin (a penny is fine), a ruler that can measure centimeters, tape, and a stopwatch

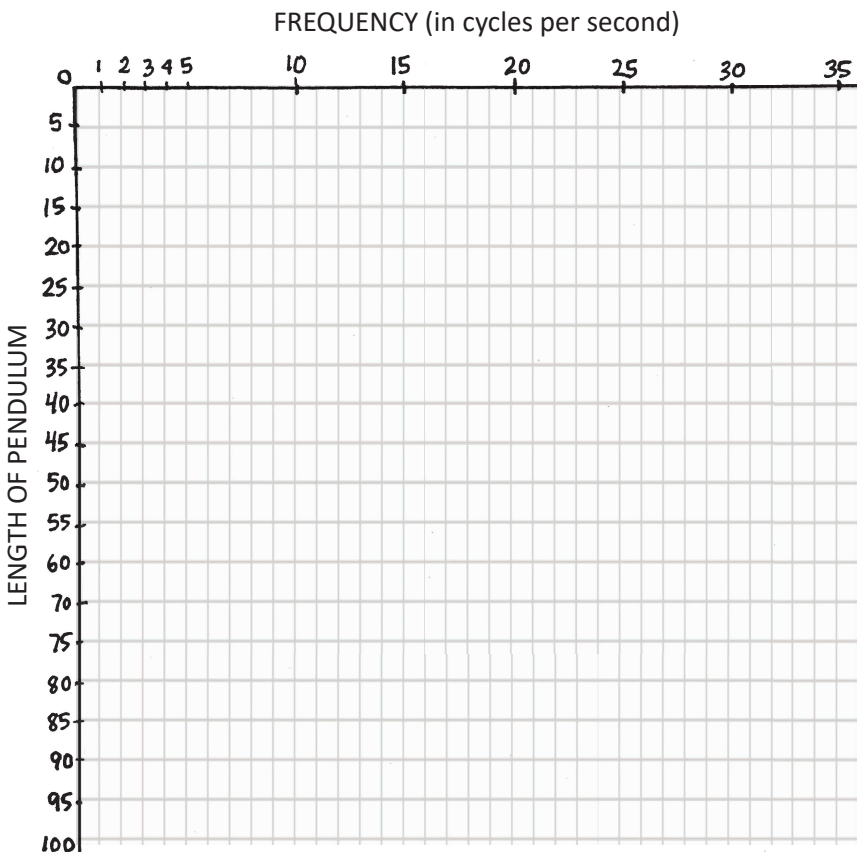
- 1) Tape the coin to the end of the thread.
- 2) Pinch the thread at precisely 5 cm above the coin. (You can make a small dot of ink on the thread if you find it helpful.)
- 3) Time this pendulum's cycles for 15 seconds. (One cycle is "out and back.") You might want to do this several times so that you can be sure of your number. Record your answer below, where it says 5 cm \_\_\_\_\_.
- 4) Now pinch the thread at exactly 20 cm above the coin. This pendulum is 4 times as long as your first one. Time it for 15 seconds and record your answer below.
- 5) Pinch the thread at exactly 45 cm above the coin. This pendulum is 9 times as long as the original. Time it for 15 seconds and record the results below. Your answer might not be a whole number. You might get something like 12.5.
- 6) Finally, pinch the thread at exactly 80 centimeters above the coin. Time it for 15 seconds. Again, your answer might not be a whole number.



5 cm \_\_\_\_\_      20 cm \_\_\_\_\_      45 cm \_\_\_\_\_      80 cm \_\_\_\_\_

7) Can you find a pattern in this number sequence? What would the frequency (number of cycles) be if the pendulum was extended to 125 cm? \_\_\_\_\_ To 180 cm? \_\_\_\_\_ What about 50 cm? \_\_\_\_\_

8) Plot your results on this graph. The points will make a distinct shape. Make a smooth "best fit" curve along the path of these points. After you make the curve, you will see that the curve can be used to estimate the frequency for any length. Put your pencil on any length number, then move the pencil to the right until it bumps into the curve. Then look up and see what frequency number is right above your pencil point.



#### WHAT YOU SHOULD NOTICE:

When the length is increased to be 4 times longer, the frequency slows to 1/2 the rate. When the length is increased to be 9 times longer, the frequency slows to 1/3 the rate. When the length is increased to be 16 times longer, the frequency slows to 1/4 the rate.

This is called the "**Inverse Square Law.**" When the length is increased N times, the frequency decreases by 1/√N. (One divided by the square root of N.) If you extended the pendulum to 125 cm, that would mean that you had made it 25 times as long. The square root of 25 is 5, so the frequency would be 1/5 of the original rate. If you shortened the pendulum to 50 cm, you would have multiplied the length by 10, which is not a perfect square number. The math is a little messier, but you can still calculate the new frequency. Find the square root of 10, (about 3.16). The frequency would be 1/3.16 times slower. Just divide your original frequency by 3.16.

### ACTIVITY 3.7: Use a pendulum to calculate a square root

*You will need: your pendulum from activity 3.6*

- 1) Tie a knot anywhere in the thread of your pendulum. Hold this knot and time the pendulum.
- 2) Fold the thread in half so that the knot touches the middle of the coin. Hold the thread by this halfway point and time this half-length pendulum.
- 3) Divide the frequency of the first pendulum by the frequency of the half-pendulum.
- 4) Use a calculator to find the square root of 2. Compare this to your pendulum calculation. Are they close?
- 5) Try it again using a knot in a different place. Same result?



We now turn some dials on our S.N.O.O.P. machine and go forward in time. We stop in Paris, France, in 1851, where a man named **Léon Foucault** (*fu-ko*) has attached a very long pendulum to the ceiling of the Panthéon. But before we turn on the S.N.O.O.P. machine, here is some information about the building.

You may have heard of the original Pantheon in Rome, which was built around the year 25 BC and dedicated to many of the gods that the Romans worshiped at that time. (It is now a Christian church.) The Panthéon in Paris was commissioned by King Louis XV in 1758 and was completed in 1790, about the time that the French revolution began. The Paris Panthéon was built as an upgrade to the Church of St. Genevieve (a female patron saint of Paris). The columns in the front are modeled after ancient Greek temples.



Pantheon in Rome

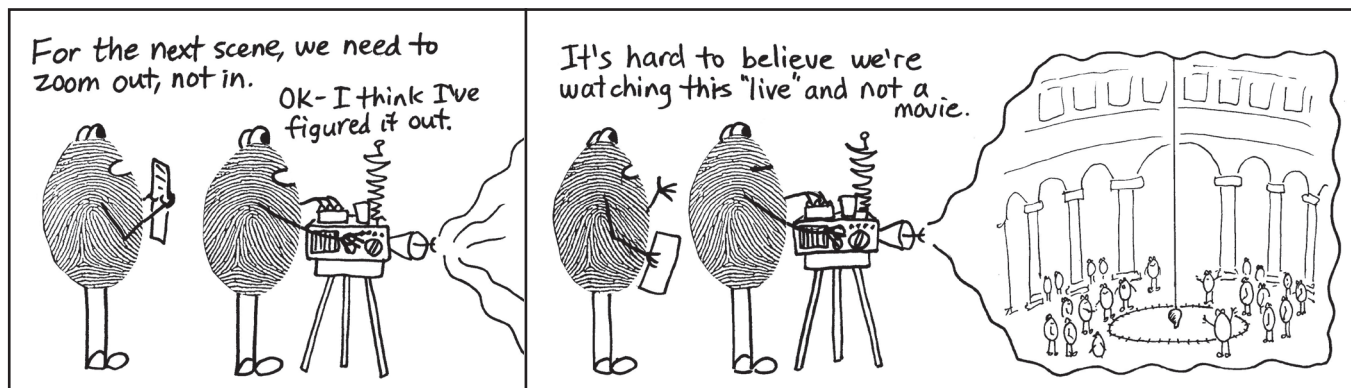


Panthéon in Paris (a 1791 painting)

The Panthéon in Paris has a very high dome. Here is what you'd see if you stood inside and looked up at the ceiling. There is a beautiful painting at the very top of the dome. You can see the blue colors of the sky in the painting.



Somehow or other, Foucault managed to convince the people in charge of the Panthéon to let him hang a pendulum from the top of that dome. (He had done this experiment in his lab and already knew it would work.)

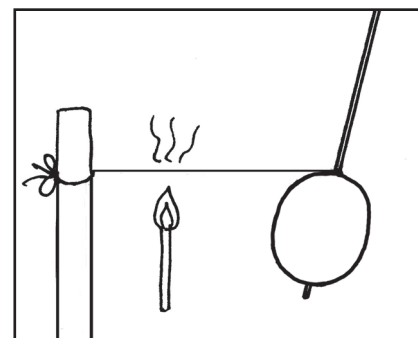


As we learned in previous activities, the length of a pendulum is what controls how fast it swings. The rope hanging from the dome was very long, so the pendulum swung very, very slowly. Foucault's reason for hanging such a long pendulum was to demonstrate that the earth rotates, spinning on its north-south axis. Foucault made marks on the floor in a large circle around the swinging pendulum. The marks would keep track of the direction in which the pendulum was swinging. The bob on the pendulum was extremely heavy, and this helped the pendulum to keep going for many hours. Over the course of the day, it looked like the pendulum was changing direction with respect to the marks on the floor. Foucault asserted that it was not the pendulum that was moving, but the earth.



This demonstration would have been easier to understand if the pendulum had been hanging over the North Pole. You can imagine the earth spinning on its axis, rotating underneath the pendulum. Because the earth is turning counterclockwise with respect to the North Pole, it would look like the pendulum was going clockwise. But Foucault's pendulum was not at a pole; it was in Europe. Why did it still trace out a circle? As long as the pendulum is not on the equator, it will trace out a circle, though more slowly than at the poles. At the poles, it would look like the pendulum traced a circle in 24 hours. Away from the poles, it takes longer than 24 hours. At the equator, it does not go in a circle at all, just back and forth.

One problem Foucault had in setting up the demonstration was that he needed the pendulum to go back and forth in a straight line, with no movement side to side. Even the slightest movement in a sideways direction would start the pendulum going in a long oval instead of a straight line. If he let the bob go with this hands, he would always accidentally introduce some sideways motion. So he devised an ingenious starting mechanism that used a match to cut a string that was pulling the pendulum back. When the match burned through the string, the pendulum was released. (When they tried this experiment at the South Pole, they were not allowed to use any fire, so no match for starting the pendulum. This made the experiment harder to do.)



Foucault pendulums on display in public buildings use electromagnets to help them keep going. Even pendulums with very heavy bobs will eventually lose energy, slow down, and stop. Pendulums in public spaces need to keep going continuously, so their swings are given a tiny extra boost by the clever use of magnetism generated by electricity. These devices can also counteract any tendency of the pendulums to start swinging in an oval pattern instead of straight back and forth.

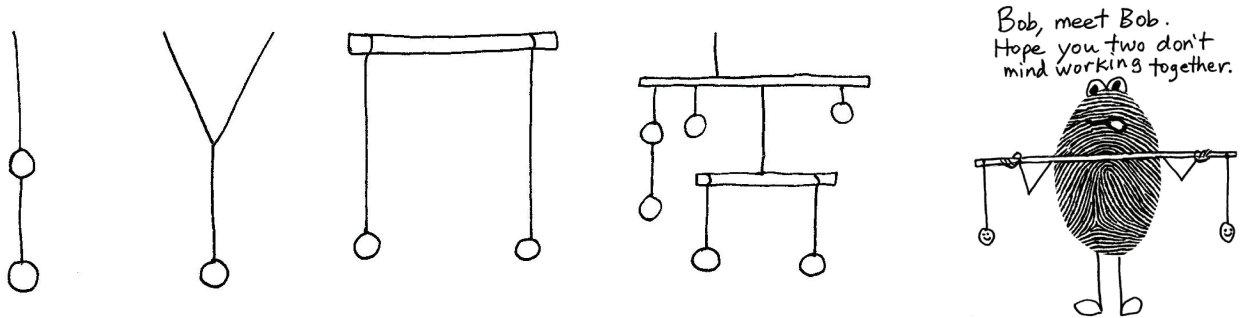
### ACTIVITY 3.8: Watch videos on the Foucault pendulum

You can access videos about Foucault pendulums by using the YouTube playlist for this curriculum.



So far, we have been working with single pendulums. What would happen if we joined two or more pendulums together? Each pendulum would have its own distinct motion, but its motion would also be affected by the motion of any other pendulum to which it was attached. What kind of motion would result? Anything interesting, or just a mess?

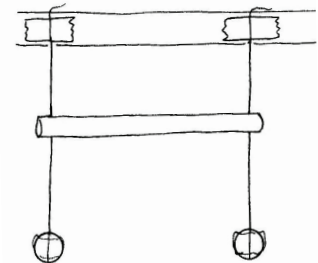
In this next activity, you will experiment with a compound pendulum. A compound pendulum is made of two or more pendulums attached together in some way. Compound pendulums come in many shapes. Mobiles (hanging sculptures) qualify as compound pendulums, although they usually aren't designed for motion.



### ACTIVITY 3.9: Pendulums that take turns

**You will need:** some thread, two coins, tape, a drinking straw, scissors

- 1) Cut two pieces of thread about 40 centimeters (16 inches) long.
- 2) Tape a coin to the end of each thread.
- 3) Tape the free ends of the threads to the edge of a table. The distance between the threads should be just a little less than the length of the straw.
- 4) Make snips in both ends of the straw. (If it has a flexible part, trim that off.) Don't cut a "V," just a straight slit.
- 5) Slip the notches of the straw onto the thread to make a bar that goes between the threads, as shown. The straw should at least be several inches (8-10 cm or so) above the coins. If the straw is too close to the coins, it will be too difficult to observe the phenomenon we want to observe.



- 6) Start one coin swinging but don't touch the other one. Watch for a while and see what happens. It should look like the pendulums are passing their energy back and forth. When one pendulum reaches the height of its energy, the other one will be at rest, and for a few seconds it won't move at all.
- 7) You can adjust the height of the straw by sliding the threads through the notches. Try the experiment again with the straw up high, then down low.

#### CAN YOU "COMMUNICATE" WITH YOUR COINS?

- 1) Re-tape your pendulums so that one is substantially longer than the other. Keep the straw positioned between the threads, and straight across (parallel to the floor).
- 2) Tap or lightly push the straw in a way that makes only one coin swing. You'll have to discover how to do this on your own. Just do a little experimenting.
- 3) Now see if you can do the reverse, getting the other coin to swing while the first one stays still.

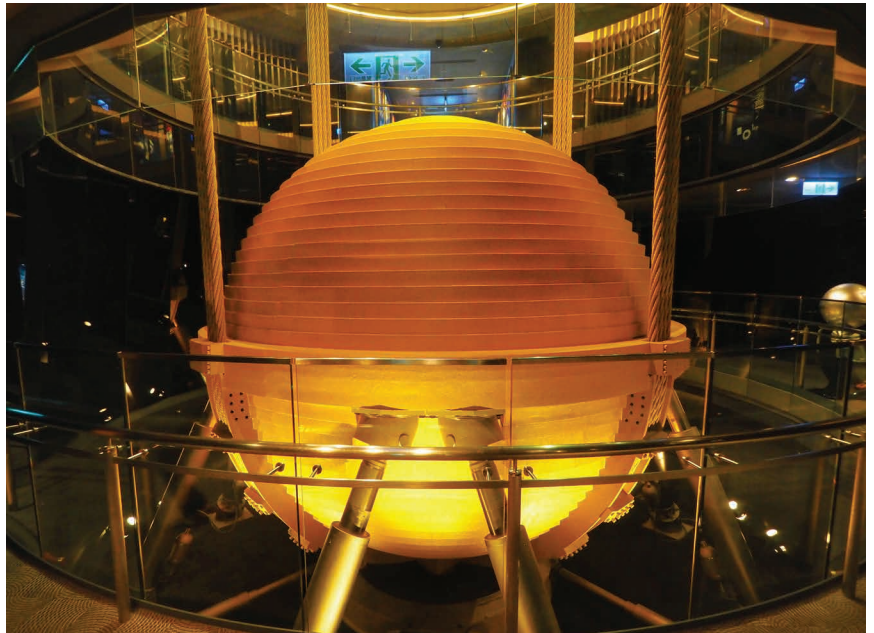
Here is the explanation of what is going on:

To get one of the coins to start swinging, you have to make your taps match the natural period of the pendulum. It's almost if as each pendulum has a narrow range of "hearing" and can only "hear" taps that match the frequency at which it would swing if it was in motion. The pendulums "ignore" taps that are not at that frequency.

A word often used to describe this phenomenon is "**resonance**." When something "resonates," it is responding to its natural frequency. The classic example of this is an opera singer shattering a glass. The note sung by the singer just happens to be at the natural frequency of the glass, so the glass "hears" the note and starts to vibrate. Then it vibrates too much and shatters. The word resonate has a broader meaning, also. If someone says, "That resonates with me," they mean that what someone else just said matches the way they, themselves, are feeling or thinking. The other person's thoughts lined up with their own thoughts.

It was fun to experiment with resonance in our compound pendulum activity. The principle of resonance has an interesting practical application in architecture. Skyscrapers can sway back and forth during wind storms, acting like upside-down pendulums. This can be very scary if you are in the top floor when the swaying starts! Theoretically, if the winds are strong enough (such as in a hurricane) they could get a building moving back and forth fast enough that the steel structure begins ripping apart. However, architects have developed a way to prevent too much swaying.

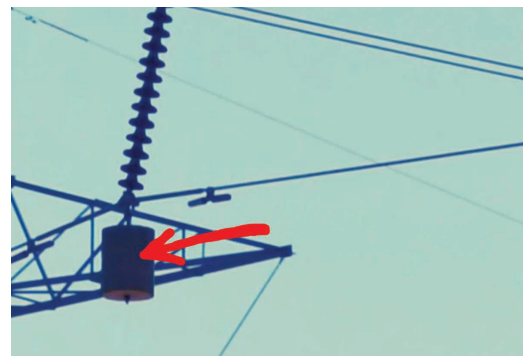
In the world's tallest buildings, the top floors have a vertical shaft in the center where a large pendulum hangs. As you might guess, the bob of this pendulum weighs many tons. Architects use mathematical formulas to determine how much weight to use, how long to make the rod, and exactly where to place it in the building. These formulas take into account how tall the building is, how wide it is, and the strength and flexibility of the materials from which it is built. They "tune" the pendulum to resonate with the building. When the building starts to sway, the motion of the building begins to be transferred to the pendulum. As the motion is transferred to the pendulum, the building sways less. Even if some of the energy of the



*The tuned mass damper in Taipei 101, in Taiwan*

pendulum gets transferred back into the building, it will be far less than the original amount of energy. Part of the design, however, is to prevent energy from going back into the building. It's amazing that something so much smaller than the building can stop it from swaying.

The pendulums in skyscrapers are called **tuned mass dampers**. To dampen something is to decrease its energy or activity level. Tuned mass dampers don't have to look like pendulums, however, and they are found in other places, not just tall buildings. Sometimes they look like cylinders or blocks that shift back and forth. Here are tuned mass dampers on power lines, helping the wires not to sway in the wind.



Bridges also use tuned mass dampers to reduce their movement. Dampers on bridges can be designed to move against the motion of the bridge, thus preventing resonance. Bridge dampers are usually some type of spring mechanism attached to a large weight. It is said that when armies march across bridges they are told to stop marching and just walk, to prevent any chance that the bridge might resonate to the frequency of the footsteps.

### **ACTIVITY 3.10: Watch a few short videos about tuned mass dampers**

*There are several videos about tuned mass dampers in the YouTube playlist. Now would be a great time to watch them!*



You're probably almost ready to leave the topic of pendulums and move on to something else, but we need to cover one more major concept. Pendulums belong to a group of mechanisms called **oscillators**. To oscillate is to "vary in magnitude or position in a regular manner about a central point." This definition is broad enough that it can be applied to many kinds of motion, as well as to situations where something shrinks and grows. The central point doesn't have to be literal, it can be a point on a graph or a point in time. For example, some people think that the universe might go through cycles of expansion and contraction. This idea is called the oscillating universe. Sound waves oscillate, and the machine that measures these oscillations is called an *ound it*. Then trace it again to make a second circle. Cut out these large cardboard circles. Make sure to puncture (shown here).



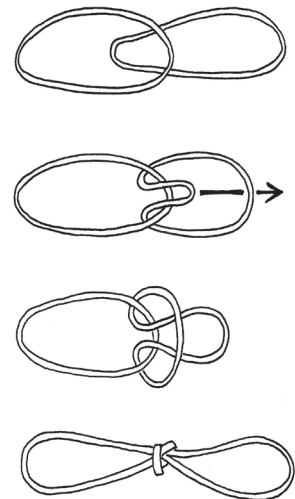
Pendulums don't have to swing back and forth; they can also go up and down. A heavy ball bouncing up and down on the end of a spring can be called a "vertical pendulum." Vertical pendulums have many of the same characteristics as swinging pendulums. But instead of reading about them, why don't you discover their characteristics for yourself?

### ACTIVITY 3.11: A pendulum that bounces instead of swings

**You will need:** at least a dozen rubber bands (thin ones, if possible, and all exactly the same size, and as new as possible since rubber bands lose elasticity as they age), your paper clip hook, a bottle of water (at least 335 ml, but no more than 500 ml), a stopwatch, a pencil, and a way to hang your vertical pendulum from someplace near the ceiling (or perhaps from the top of an open door)

#### PREPARATION:

- 1) Join about ten rubber bands, end to end, as shown in diagram on right. Thin ones are better than thick ones, but size is less important than making sure all the bands are identical in size and stretchiness. (Avoid old rubber bands, as age makes them brittle.)
- 2) Find a way to hang this rubber band chain from a very high place so that the bottom of the chain is several feet off the floor. The chain should be able to stretch down to the floor.
- 3) Put a rubber band around the neck of the water bottle. Wrap it several times so it is tight. Then put your paper clip hook onto it and position the hook so that you can hang the bottle from one of the rubber bands.
- 4) Hang the bottle from the bottom band. If the hanging bottle touches the floor, you'll need to either increase the height of the chain, or decrease the number of rubber bands in the chain, or decrease the amount of water in the bottle. The bottle needs to be able to go up and down.



#### NOTES ABOUT DOING THE EXPERIMENTS:

These general directions might have to be adjusted to your situation. Your rubber bands might be smaller or larger than average, for example. If you need to use more or less than 10 rubber bands, that's fine—you don't need to use all the squares on the data chart. Also, you may have trouble counting the number of cycles (up and down) towards the end of the 10 seconds. Just to the best you can. Keep the rhythm in your head as the bottle slows down. You also might need to use fractions in your answers. For example, a bob might complete 6.5 cycles in 10 seconds.  
**TIP:** It helps if you have two people to do these experiments, one to watch the clock and one to watch the bottle. If you are working on your own, hold the stopwatch right near the bottle so you can see both.

- 5) Hook the bottle onto the lowest rubber band. Start the stopwatch and count cycles for 10 seconds. Record the number of cycles on the data chart on the next page.
- 6) Move up the chain to the next highest rubber band. Count cycles for another 10 seconds. Record data.
- 7) Continue up the rubber band chain. You might have to stop at about 3 from the top. It can be very difficult or impossible to time just one rubber band.
- 8) Now adjust the amount of water in the bottle by pouring out half of it.
- 9) Start back down at your lowest rubber band and count cycles for 10 seconds. Record data. Continue up the chain.
- 11) After you have recorded all your data in the chart, plot each point on the graph.
- 12) Connect the dots by drawing a curve or a line that doesn't necessarily go through every point, but shows the general trend.

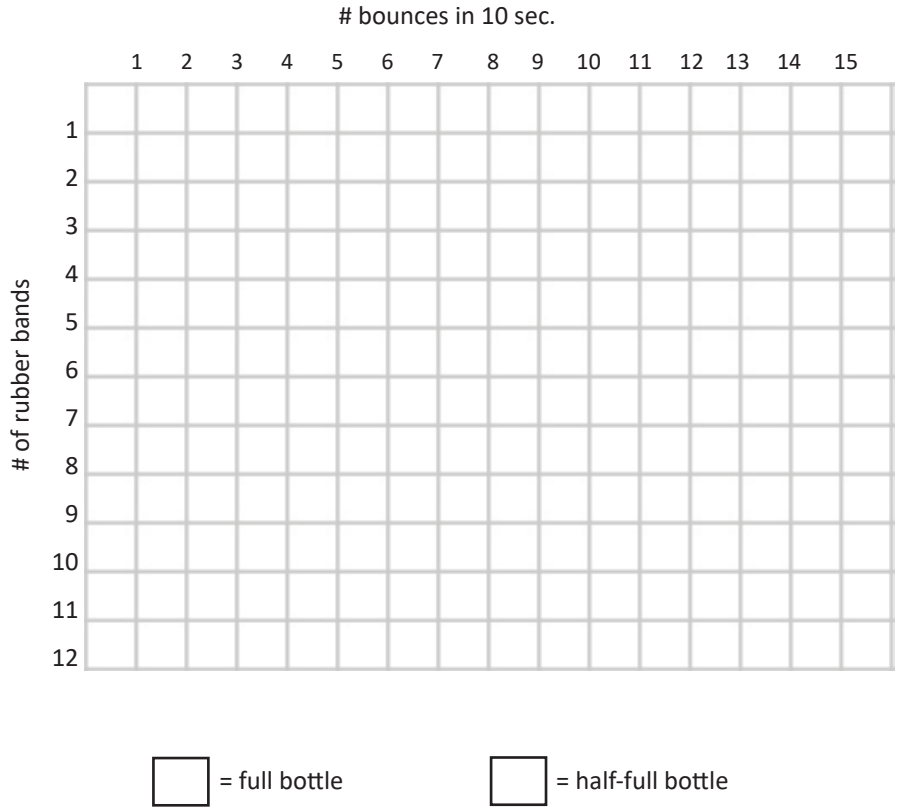
DATA CHART:

# bounces in 10 sec.

1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

↑ full bottle      ↑ half-full bottle

GRAPH YOUR DATA:



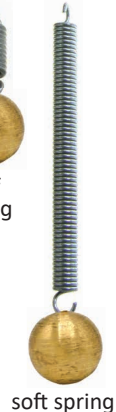
Follow-up questions:

- 1) Which bounced longer, the heavier bottle or the lighter bottle? \_\_\_\_\_
- 2) In your thread pendulums, which swung for a longer time, 1 clip or 10 clips? \_\_\_\_\_
- 3) Did the bottles seem to come to rest more quickly than your thread pendulums? \_\_\_\_\_
- 4) A longer rubber band chain produced (greater/fewer) number of cycles per 10 seconds. (circle one)
- 5) Did the weight of the bottle seem to affect the length of the cycles? \_\_\_\_\_

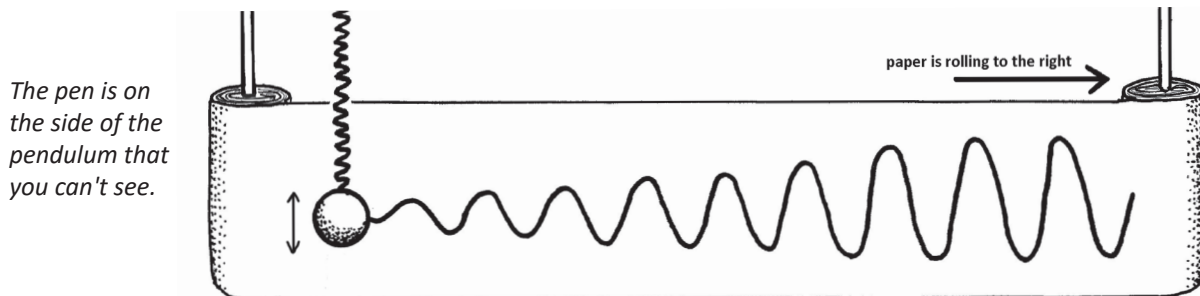
Extend the lines/curves you drew on your graph so that they go beyond the edges a bit. This will let you do something called **extrapolation**. This is a very important tool in scientific research. It lets scientists make reasonable guesses about what would have happened at points above and below their actual data points. If you were only able to use 10 rubber bands, what would have happened if you had been able to use 12 or 13? If you extend the end of your line, taking it below the graph to where 13 or 14 would be, you can see where the line crosses the vertical lines that indicate cycles per 10 seconds. Use extrapolation to answer these questions:

- 6) For the lighter bottle, estimate the number of cycles per 10 seconds for:  
 13 bands: \_\_\_\_\_      14 bands: \_\_\_\_\_      15 bands: \_\_\_\_\_
- 7) For the heavier bottle, use extrapolation to estimate the number of cycles per 10 seconds for:  
 2 bands: \_\_\_\_\_      1.5 bands (notice decimal point!): \_\_\_\_\_

Your patience with watching vertical pendulums is probably wearing thin, so we'll just do a thought experiment to conclude our work on this topic. Imagine a bob hanging from a very stiff spring that is hard to pull. Give that imaginary bob a pull downwards and let it go. It snaps back almost instantly. The bob didn't really go up and down at all. Now imagine a spring that is very easy to stretch. Perhaps it is very long. Put a bob on the end of that "soft" spring and give it a pull downwards. The soft spring allows the bob to pull it down a considerable distance. It then slowly shrinks, taking the bob on a very gentle ride upwards. Then all the way down again. The bob gets a very long ride as the soft spring stretches out again and again. So we can see that with vertical pendulums, other factors are important, such as the size and the stiffness of the spring. Our thread pendulums didn't have any stiffness issues; we only had to consider the length of the thread.



Now let's put a pen in the bob of a vertical pendulum and a roll of paper behind it. We'll pull out the paper at a steady rate and let the pen in the bob touch the paper. The shape traced onto the paper will be a gently rolling wave shape. (There is a video of this on the playlist. You might want to watch it right now.) As the vertical pendulum experiences dampening due to friction and to heat generated in the spring, and begins to run out of energy, the height of the waves will decrease, but the length of the waves will remain the same. (The length of the wave is the frequency of the cycles, which does not change.)

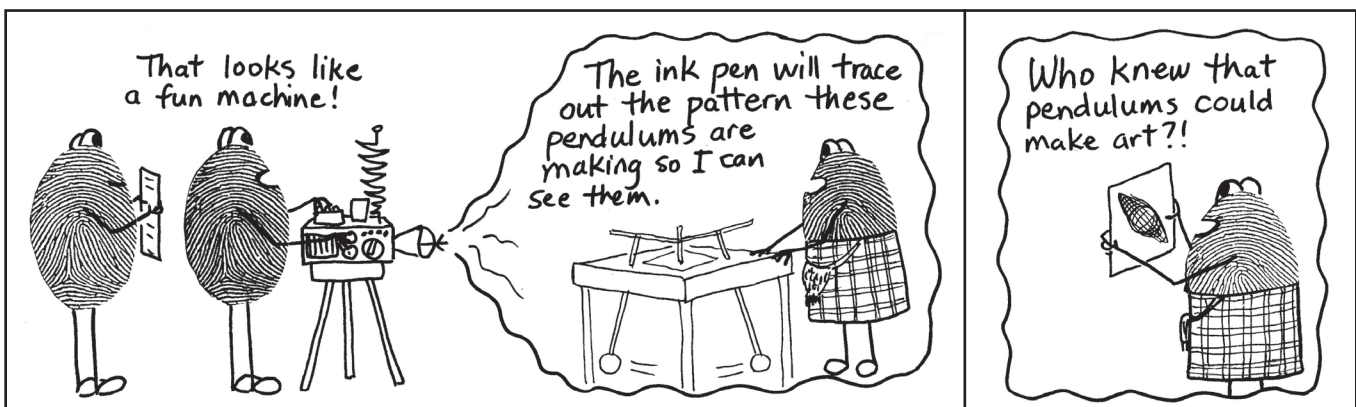


Physicists are very aware of the connection between up-and-down oscillations and this wave pattern. The wave is called a **sinusoidal** (*sine-yu-soy-dul*) wave, or just **sine** wave. Sine waves are a mathematical way to record oscillating motion. There are many equations associated with sine waves, such as the one shown here.

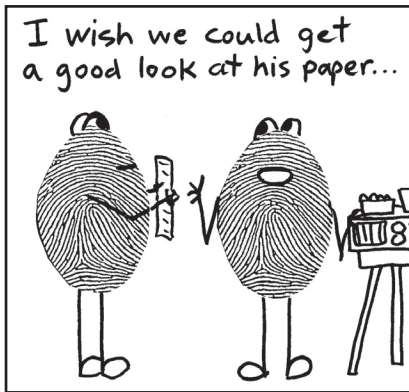
$$x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}}t\right) + \frac{v_0}{\sqrt{\frac{k}{m}}} \sin\left(\sqrt{\frac{k}{m}}t\right)$$

We need one more vocabulary word to finish this discussion. Physicists often add the word "harmonic" to this type of oscillation, and call it **harmonic oscillation**. Pendulums are called **simple harmonic oscillators**. You might recognize the word "harmonic" (and the related word "harmony") as musical terms. Sound waves are oscillations of air molecules. The pitch (high or low) is related to the length of the wave (distance from peak to peak).

Our last stop with the S.N.O.O.P. machine for this chapter will be Scotland in 1844. We will meet a mathematician named Hugh Blackburn who is inventing a machine that can record the harmonic oscillations of two pendulums that are at right angles.





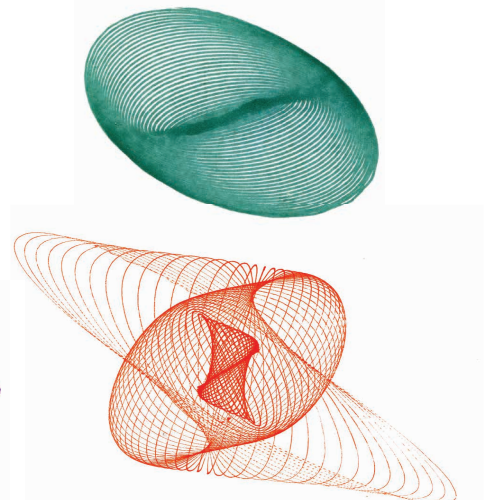
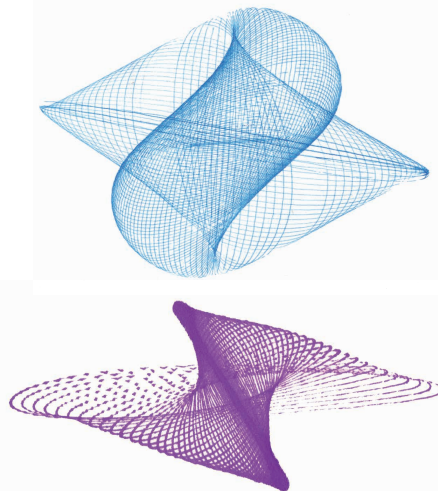
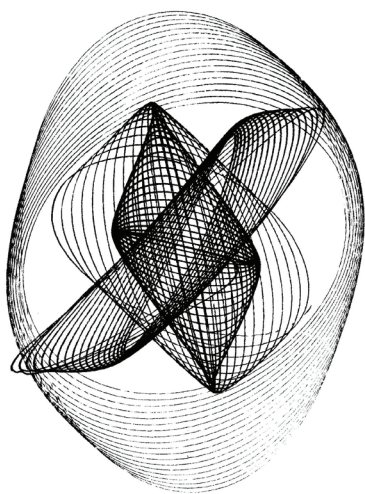
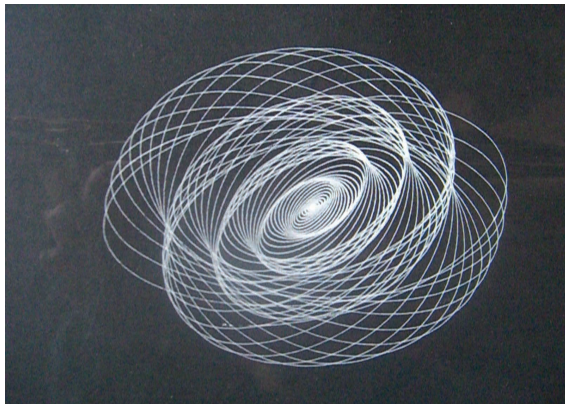


The readers would also like to see what Hugh is holding. We don't have any of the images that his machine drew, nor any details about what his machine actually looked like, so we'll have to use modern images that are probably very similar. Hugh Blackburn's work inspired other mathematicians and physicists to do more investigations, and as a result, several types of **harmonographs** have been developed. Hugh's machine likely only had two pendulums, but the most popular harmonograph today (judging by the number of YouTube videos about it) is the "three-pendulum rotary harmonograph." The word "rotary" refers to something that goes around in circles. One of the three pendulums swivels around a pivot point, instead of going back and forth.

The author of this book built her own three-pendulum rotary harmonograph, shown here in the photo. You can see the pen against the blue background. The rotary table looks like it has a sheet of black paper lying on it. Perhaps the machine is set up for making a white-on-black pattern like the one shown below. (A video of this machine in operation is on the playlist.) The paper table sitting on top of the rotary pendulum goes around in a circle while the top "arms" that are attached to the side pendulums go back and forth. The combined motion of all three pendulums creates the pattern.



The operator doesn't have much control over which design the machine will make—it seems to have a mind of its own. Although it makes similar patterns, it never does exactly the same thing twice.



All of these patterns below were drawn by the author's harmonograph. Visitors will often compare these designs to Spirograph®. The main difference is that with a harmonograph, you have a dampening effect as the pendulum loses energy and slows down. This means that the drawing spirals inward and never retraces the same path. With Spirograph, once the pattern is established, the pen traces over the same lines it again and again and again.

**ACTIVITY 3.12: Review/quiz** (Answers are in the teacher's section.)

TRUE or FALSE?

- 1) \_\_\_ The weight of the bob is what controls the period of the pendulum.
- 2) \_\_\_ Pendulum cycles for very large swings are a tiny bit faster than for very small swings.
- 3) \_\_\_ Galileo's father was very supportive of his son's interest in math.
- 4) \_\_\_ Foucault hung a pendulum from the dome of the Pantheon in Rome.
- 5) \_\_\_ Foucault wanted to demonstrate the rotation of the earth.
- 6) \_\_\_ Euclid's geometry book was only used in ancient Greece.
- 7) \_\_\_ Foucault discovered the Inverse Square Law.
- 8) \_\_\_ The Inverse Square Law lets you predict what will happen to frequency if you increase length.
- 9) \_\_\_ Air molecules have no effect on swinging pendulums.
- 10) \_\_\_ Pendulums can bounce as well as swing.

Fill in the blanks.

- 11-12) Name two places you might find a "tuned mass damper." \_\_\_\_\_ and \_\_\_\_\_
- 13) Galileo watched a chandelier in the cathedral of this city: \_\_\_\_\_.
- 14) A \_\_\_\_\_ wave is how mathematicians represent oscillating motion.
- 15) If you want to speed up your pendulum you should \_\_\_\_\_.
- 16) This is when you extend data lines above and below your experimental results: \_\_\_\_\_
- 17) When something vibrates at its natural frequency we say that it \_\_\_\_\_.
- 18) To vary in magnitude or position in a regular manner about a central point: \_\_\_\_\_
- 19) The \_\_\_\_\_ of a pendulum is the number of cycles per second.
- 20) Foucault pendulums in modern public buildings use \_\_\_\_\_ to keep them going.

Questions from previous chapters:

- 21) Which one of these is NOT a way to calculate mechanical advantage?
  - a) radius of wheel/radius of axle
  - b) radius of follower/radius of driver
  - c) length of lever/length of fulcrum
  - d) length of force arm/length of resistance arm
- 22) TRUE or FALSE? \_\_\_ The balance point of an object can be outside of the object.
- 23) TRUE or FALSE? \_\_\_ An idler gear has no effect on mechanical advantage.
- 24) What shape does a ball trace out in the air when you give it a toss?
  - a) oval
  - b) semi-circle
  - b) ellipse
  - d) parabola
  - e) parallelogram
  - f) parable
- 25) Which one of these is a second class lever?
  - a) seesaw
  - b) scissors
  - c) wheelbarrow
  - d) baseball bat
  - e) broom

And don't forget  
to check the playlist  
for additional pendulum  
videos!

